Computer Science 456/656 Spring 2020

Second Examination April 30, 2020

The entire examination is 205 points.

Name:____

The exam is take-home, open book, open notes, open internet. You must finish by midnight of April 30. Scan and email the completed examination paper to your TA, Pradip Marahajan. The email must have an April 30 time stamp.

1. True or False. [5 points each] T = true, F = false, and O = open, meaning that the answer is not known to science at this time.

   i ________ Every subset of a regular language is regular.

   ii ________ EXP-TIME $\subseteq$ EXP-SPACE.

   iii ________ There exists a context-sensitive language which is $\mathcal{P}$-SPACE complete.

   iv ________ Every finite language is regular.

   v ________ The language $\left\{a^i b^j a^j b^i \,:\, i,j \geq 0\right\}$ is context-free.

   vi ________ Any languge generated by an unrestricted grammar is recursively enumerable.

   vii ________ Every polynomial time language is context-free.

   viii ________ If L is in $\mathcal{P}$-SPACE, there is a reduction of L to the regular expression equivalence problem.

   ix ________ The union of two undecidable languages is always undecidable.

   x ________ The union of two recursively enumerable languages is always recursively enumerable.

   xi ________ The union of two co-RE languages is always co-RE. Hint: THINK!

   xii ________ $\mathcal{NC} = co - \mathcal{NC}$. Hint: THINK!

   xiii ________ The set of all regular expressions for regular languages over the alphabet $\{a, b\}$ is a context-free language.

   xiv ________ Various websites, such as https://www.youtube.com/watch?v=bQnjbDHefgc give solutions to various instances of RUSH HOUR. If there is a solution to a particular instance of RUSH HOUR, that solution can always be explained in polynomial time.

   xv ________ The factoring problem for an integer written in binary is both $\mathcal{NP}$ and co-$\mathcal{NP}$.

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xvi ________ If someone somewhere on the Earth publishes a correct proof that the partition problem is in $\mathcal{P}$-TIME, then it will be known that $\mathcal{P} = \mathcal{NP}$.

xvii ________ If someone somewhere on the Earth publishes a correct proof that the factoring problem for binary numerals is in $\mathcal{P}$-TIME, then it will be known that $\mathcal{P} = \mathcal{NP}$.

xviii ________ $\mathcal{NC} = \mathcal{P}$-SPACE

xix ________ co-$\mathcal{NP} \subseteq \mathcal{P}$-SPACE.

2. Fill in the blanks. [10 points each blank.]

   (a) If $L \subseteq \Sigma^*$ is $\mathcal{NP}$ time, there is a constant k and a deterministic machine V such that, for string $w \in \Sigma^*$, we have $w \in L$ if and only if there is a string c, called a ________________________ for w, such that $|c| \leq |w|^k$ and V accepts the string cw within $|w|^k$ steps.

   (b) The practicality of the RSA one-way encryption system depends on the assumption (which has not been verified) that the ________________________ problem cannot be solved in polynomial time.

   (c) $\mathcal{NC}$ is the class of languages which can be decided in ____ time using polynomially many processors.

3. [20 points] Every context-free language has a minimum pumping length. For example, the minimum pumping length of $\{a^n b^n : n \geq 0\}$ is 2.

   The language $L = \{a^n b^m cde^n : n, m \geq 0\}$ is context-free.

   (a) Find the minimum pumping length of L. Call it p. ________

   (b) For every string $w \in L$ of length at least p, there are strings u, v, x, y, z such that $w = uvxyz$ and three other conditions are satisfied. Find the strings u, v, x, y, z if $w = aabcdee$.

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4. [20 points] Give a polynomial time reduction of 3-SAT to the independent set problem.

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5. [20 points] Prove that the halting problem is undecidable.

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6. [20 points] Prove that the language $L = \{a^n b^m c^m d^n \, : \, n, m \geq 0\}$ is context-free by giving a context-free grammar for L.