1. Write a regular expression for the language consisting of all strings over \{a, b\} which contain the substring \(aaa\).

\[(a + b)^* aaa (a + b)^*\]

2. Use the method given on page 86 of the sixth edition of Linz, or on page 89 of the fifth edition, to find a regular expression equivalent to the following NFA.

\[(a + b)^* a (a + b) (a + b)\]

3. The following DFA accepts the language consisting of all binary numerals for positive multiples of three, where a leading 0 is allowed. Use the method given on page 86 of the sixth edition of Linz, or on page 89 of the fifth edition, to find an equivalent regular expression.

\[(0 + 1(01^*0)^*)^*\]

4. (a) State the pumping lemma for regular languages.

For any regular language \(L\) there exists a positive integer \(p\), called the \textit{pumping length} of \(L\), such that for any \(w \in L\) if \(|w| \geq p\) there exist strings \(x, y, z\) such that the following four conditions hold:

1. \(w = xyz\)
2. \(|xy| \leq p\)
3. \(|y| \geq 1\)
4. For any integer \(i \geq 0\) \(xy^iz \in L\)

(b) Use the pumping lemma to prove that the language \(L = \{a^nb^n : n \geq 0\}\) is not regular.

\textit{Proof:} Suppose that \(L\) is regular. Let \(p\) be the pumping length of \(L\). Let \(w = a^pb^p \in L\). Let \(x, y, z\) be strings which satisfy the four conclusions of the pumping lemma. Since \(w = xyz\) and \(|xy| \leq p\), \(xy\) consists entirely of \(a\)'s. Thus \(y = a^k\), and \(k \geq 1\). Pick \(i = 0\). Then \(xy^0z = xz = a^{p-k}b^p \in L\). Since \(p - k < p\), \(xz \notin L\), contradiction. We conclude that \(L\) is not regular. \(\qed\)
5. Work problem 9(a) on page 138 of the sixth edition, which is problem 7(a) on page 137 of the fifth edition.

Find a context-free grammar for $L = \{a^n b^n : n \leq m + 3\}$.

$$S \rightarrow T \mid aT \mid aaT \mid aaaT$$

$$T \rightarrow aTb \mid Tb \mid \lambda$$

6. Work problem 9(c) on page 138 of the sixth edition, which is problem 7(c) on page 137 of the fifth edition.

Find a context-free grammar for $L = \{a^n b^n : n \neq 2m\}$.

$$S \rightarrow aA \mid Bb$$

$$A \rightarrow aaAb \mid aA \mid \lambda$$

$$B \rightarrow aaBb \mid Bb \mid \lambda$$


Every left parenthesis is paired with a right parenthesis, and every left bracket is paired with a right bracket. Any two such pairs are either nested or disjoint.

Here is an ambiguous grammar.

$$S \rightarrow SS \mid (S) \mid [S] \mid \lambda$$

Here is an unambiguous grammar.

$$S \rightarrow (S)S \mid [S]S \mid \lambda$$

Here is another unambiguous grammar.

$$S \rightarrow S(S) \mid S[S] \mid \lambda$$


Here is an ambiguous grammar, with start symbol $E$.

$$E \rightarrow E + E \mid EE \mid (E) \mid a \mid b \mid \text{"lambda"} \mid \emptyset$$

where “lambda” means the actual symbol $\lambda$, not the empty string.

Here is an unambiguous grammar.

$$E \rightarrow T + E \mid T$$

$$T \rightarrow FT \mid F$$

$$F \rightarrow a \mid b \mid \text{"lambda"} \mid \emptyset \mid F^* \mid (E)$$

$E$ stands for “expression,” $T$ stands for “term,” and $F$ stands for “factor.”