University of Nevada, Las Vegas Computer Science 456/656 Spring 2021

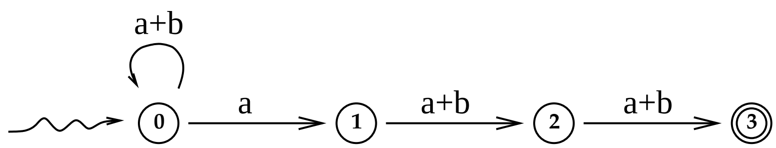
Answers to Assignment 2: Due Thursday February 11, 2021

In each case, the identical problem is in both fifth and sixth editions of your textbook.

1. Write a regular expression for the language consisting of all strings over $\{a, b\}$ which contain the substring aaa.

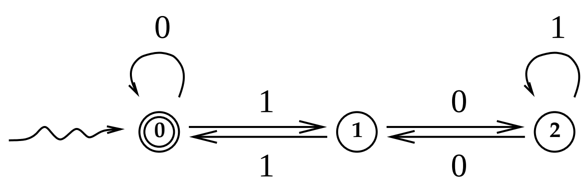
   $(a + b)^* aaa(a + b)^*$

2. Use the method given on page 86 of the sixth edition of Linz, or on page 89 of the fifth edition, to find a regular expression equivalent to the following NFA.



   $(a + b)^* a(a + b)(a + b)$

3. The following DFA accepts the language consisting of all binary numerals for positive multiples of three, where a leading 0 is allowed. Use the method given on page 86 of the sixth edition of Linz, or on page 89 of the fifth edition, to find an equivalent regular expression.



   $(0 + 1(01^*0)^*1)^*$

4. (a) State the pumping lemma for regular languages.

   For any regular language L there exists a positive integer p, called the *pumping length* of L, such that for any $w \in L$ if $|w| \geq p$ there exist strings x, y, z such that the following four conditions hold:
   1. $w = xyz$
   2. $|xy| \leq p$
   3. $|y| \geq 1$
   4. For any integer $i \geq 0$ $xy^i z \in L$

   (b) Use the pumping lemma to prove that the language $L = \{a^n b^n : n \geq 0\}$ is not regular.

   Proof: Suppose that L is regular. Let p be the pumping length of L. Let $w = a^p b^p \in L$. Let x, y, z be strings which satisfy the four conclusions of the pumping lemma. Since $w = xyz$ and $|xy| \leq p$, xy consists entirely of a's. Thus $y = a^k$, and $k \geq 1$. Pick $i = 0$. Then $xy^0 z = xz = a^{p-k} b^p \in L$. Since $p - k < p$, $xz \notin L$, contradiction. We conclude that L is not regular. ▮

5. Work problem 9(a) on page 138 of the sixth edition, which is problem 7(a) on page 137 of the fifth edition.

   Find a context-free grammar for $L = \{a^n b^n \,:\, n \leq m + 3\}$.
   $S \rightarrow T \,|aT \,|\, aaT \,|\, aaaT$
   $T \rightarrow aTb \,|\, Tb \,|\, \lambda$

6. Work problem 9(c) on page 138 of the sixth edition, which is problem 7(c) on page 137 of the fifth edition.

   Find a context-free grammar for $L = \{a^n b^n \,:\, n \neq 2m\}$.
   $S \rightarrow aA \,|\, Bb$
   $A \rightarrow aaAb \,|\, aA \,|\, \lambda$
   $B \rightarrow aaBb \,|\, Bb \,|\, \lambda$

7. Work problem 24 on page 140 of the sixth edition, which is problem 22 on page 139 of the fifth edition.

   Every left parenthesis is paired with a right parenthis, and every left bracket is paired with a right bracket. Any two such pairs are either nested or disjoint.

   Here is an ambiguous grammar.
   $S \rightarrow SS \,|\, (S) \,|, [S] \,|\, \lambda$

   Here is an unambigous grammar.
   $S- > (S)S \,|\, [S]S \, \lambda$

   Here is another unambigous grammar.
   $S- > S(S) \,|\, S[S] \, \lambda$

8. Work problem 25 on page 140 of the sixth edition, which is problem 23 on page 139 of the fifth edition.

   Here is an ambiguous grammar, with start symbol E.
   $E \rightarrow E + E \,|\, EE \,|\, (E) \,|\, a \,|\, b \,|\, "lambda" \,|\, \emptyset$
   where "*lambda*" means the actual symbol λ, not the empty string.

   Here is an unambiguous grammar.
   $E \rightarrow T + E \,|\, T$
   $T \rightarrow FT \,|\, F$
   $F \rightarrow a \,|\, b \,|\, "lambda" \,|\, \emptyset \,|\, F^* \,|\, (E)$

   E stands for "expression," T stands for "term," and F stands for "factor."