University of Nevada, Las Vegas Computer Science 456/656 Spring 2021

Answers to Assignment 3 due Tuesday March 2, 2021

Name:____

1. True or False. 5 points each. T = true, F = false, and O = open, meaning that the answer is not known science at this time.

   (a) **F** The context-free grammar equivalence problem is decidable.

   (b) **T** The context-free grammar equivalence problem is co-$\mathcal{RE}$.

   (c) **T** If L_1 is a regular language and L_2 is a context-free language, then $L_1 \cap L_2$ is context-free.

   (d) **T** If there is a recursive reduction of L_1 to L_2, where L_1 is an undecidable language, then L_2 must be undecidable.

   (e) **O** The factoring problem is in $\mathcal{P}$.

   (f) **T** If L is a recursive language, there must be a machine which enumerates L in canonical order.

   (g) **T** If there is a machine which enumerates a language L in canonical order, then L must be recursive.

2. State the pumping lemma for regular languages.

   If L is a regular language, there is an integer p such that for any $w \in L$, if $|w| \geq p$, there exist strings x, y, z such that:

   (a) $w = xyz$

   (b) $|xy| \leq p$

   (c) $|y| \geq 1$

   (d) For any integer $i \geq 0$, $xy^iz \in L$

3. Use the pumping lemma to prove that the Dyck language is not regular.

   Let D be the Dyck language, and suppose D is regular. Let p be the pumping length of D, and let $w = a^pb^p$, which is a member of D of length at least p. Then there exist strings x, y, z such that

   (a) $a^pb^p = xyz$

   (b) $|xy| \leq p$

   (c) $|y| \geq 1$

   (d) For any $i \geq 0$, $xy^iz \in D$.

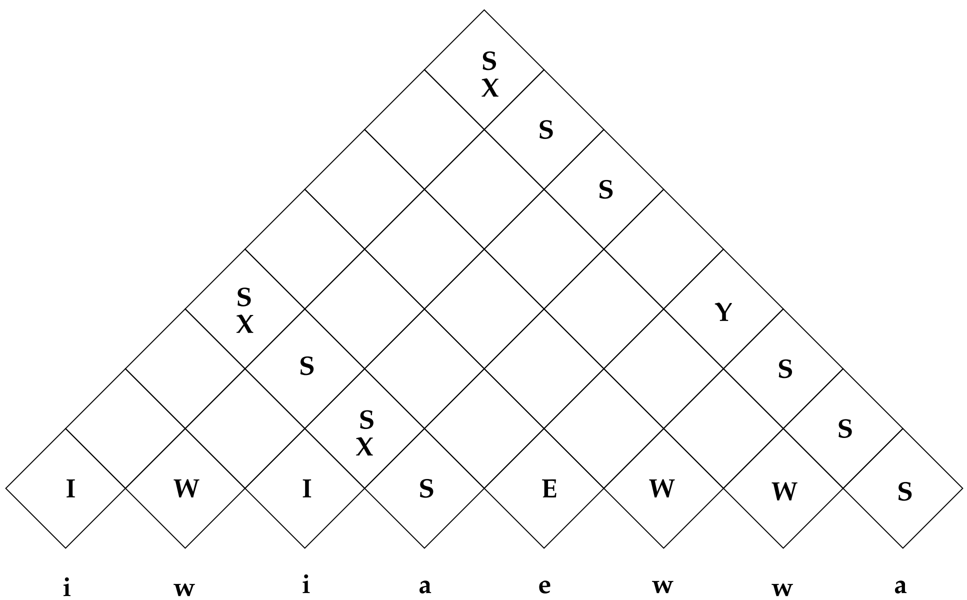
   Note that $|w| = 2p$. From (a) and (b), we have that xy is a substring of the first p symbols of w, hence $y = a^k$ for some $k \leq p$. From (c), we conclude that xy is not the empty string, hence $k \geq 1$. Let $i = 0$. From (d), we conclude that $xy^0z = a^{p-k}b^p \in D$, which is impossible since every member of D has the same number of a's as b's. Contradiction. Thus D is not regular.

4. Consider the Chomsky Normal Form grammar G given below.
$S \to IS$
$S \to WS$
$S \to XY$
$X \to IS$
$Y \to ES$
$S \to a$
$I \to i$
$W \to w$
$E \to e$

(a) Show that G is ambiguous by giving two different **leftmost** derivations for the string *iiaea*.

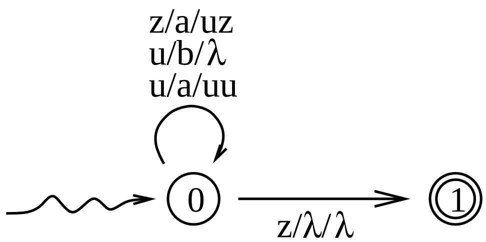
$S \Rightarrow IS \Rightarrow iS \Rightarrow iXY \Rightarrow iISY \Rightarrow iiSY \Rightarrow iiaY \Rightarrow iiaES \Rightarrow iiaeS \Rightarrow iiaea$
$S \Rightarrow XY \Rightarrow ISY \Rightarrow iSY \Rightarrow iISY \Rightarrow iiSY \Rightarrow iiaY \Rightarrow iiaES \Rightarrow iiaeS \Rightarrow iiaea$

(b) Use the CYK algorithm to prove that $iwiaewwa \in L(G)$.



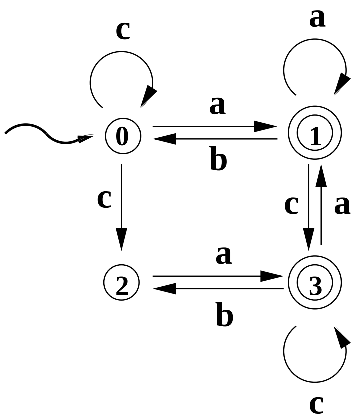
$iiaea \in L(G)$ because $V_{1,n}$ contains the start symbol, where $n = |iwiaewwa| = 8$

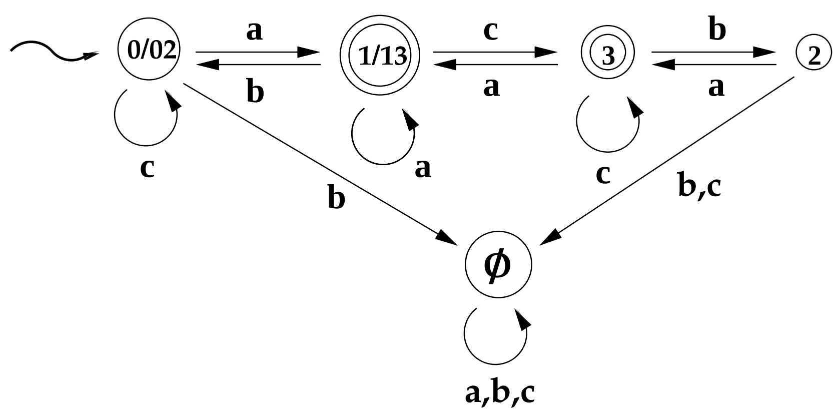
5. Let L be the language accepted by the PDA diagrammed below. What is L? You can either describe L in a **few** words, or give a context-free grammar for L.



L is the Dyck language, with context-free grammar
$S \to aSbS \mid \lambda$

6. Find a minimal DFA equivalent to the NFA shown below.





	a	b	c
0	1	$\emptyset$	02
1	1	0	3
3	1	2	3
2	3	$\emptyset$	$\emptyset$
02	13	$\emptyset$	02
13	1	02	3

Of the sixteen subsets of $Q = \{9, 1, 2, 3\}$, seven are reachable. The empty set is the dead state.
$\{0\}$ and $\{0, 2\}$ are equivalent, and $\{1\}$ and $\{1, 3\}$ are equivalent. Thus, the minimal DFA has five states, including the dead state.

7. Give a context-sensitive grammar for $\{a^n b^n c^n d^n \; : \; n \geq 1\}$

There are infinitely many correct answers. This one is not the shortest, but it is easy to understand.

$S \to abcd \mid Aabcd$
$Aa \to aaB \mid AaaB$
$Ba \to aB$
$Bb \to bbC$
$Cb \to bC$
$Cc \to ccD$
$Dc \to cD$
$Dd \to dd$

A is a messenger, which doubles the first a then changes to B, which doubles the first b then changes to C, which doubles the first c then changes to D, which doubles the first d and then disappears.
A optionally generates a new messenger, so the doubling can occur any number of times.

8. Fill in the following table, showing which operations are closed for each class of languages. In each box, write **T**, **F**, or **O**.

language class	union	intersection	concatenation	Kleene closure	complementation
$\mathcal{NC}$	T	T	T	T	T
regular	T	T	T	T	T
context-free	T	F	T	T	F
$\mathcal{P}$	T	T	T	T	T
$\mathcal{NP}$	T	T	T	T	O
co-$\mathcal{NP}$	T	T	T	T	O
recursive	T	T	T	T	T
$\mathcal{RE}$	T	T	T	T	F
co-$\mathcal{RE}$	T	T	T	T	F
undecidable	F	F	F	F	T

9. Prove that the halting problem is undecidable.

The proof is by contradiction. Let L_{HALT} be the language of all $\langle M \rangle x$ such that M halts with input x. Assume that the L_{HALT} is decidable.

Let $L_D = \{\langle M \rangle \; : \; \langle M \rangle \langle M \rangle \notin L_{HALT}\}$, the *diagonal language*. L_D is decidable since L_{HALT} is decidable. Let M_D be a machine which decides L_D.

Since M_D accepts L_D, $\langle M_D \rangle \in L_D$ if and only if M_D halts with input $\langle M_D \rangle$.

By definition of L_D, $\langle M_D \rangle \in L_D$ if and only if M_D does not halt with input $\langle M_D \rangle$.

Contradiction. Thus, L_{HALT} is not decidable.