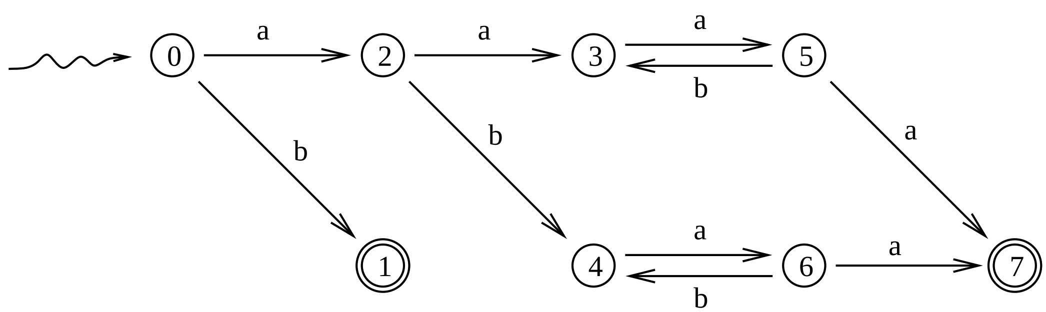
Answers to Practice for the Final Examination: Part I

1. Design a minimal DFA equivalent to the DFA shown below.



The start state is 0, the only final state is $\{1, 7\}$, $\emptyset$ is a dead state, and the transition table is:

	a	b
$\{0\}$	$\{2\}$	$\{1, 7\}$
$\{1, 7\}$	$\emptyset$	$\emptyset$
$\{2\}$	$\{3, 4\}$	$\{3, 4\}$
$\{3, 4\}$	$\{5, 6\}$	$\emptyset$
$\{5, 6\}$	$\{1, 7\}$	$\{3, 4\}$

2. Let L be the language consisting of all strings over the binary alphabet which contain the substring 011. Design a minimal DFA which accepts L.

There are four states, 0, 1, 2, and 3. The start state is 0 and the only final state is 3. The transition table is:

	0	1
0	1	0
1	1	2
2	1	3
3	3	3

3. Design a DPDA which accepts the language over $\{a, b\}$ consisting of all strings which have equal numbers of each symbol. (For example, *ab*, *ba*, *abba*, and *abbaab*.)

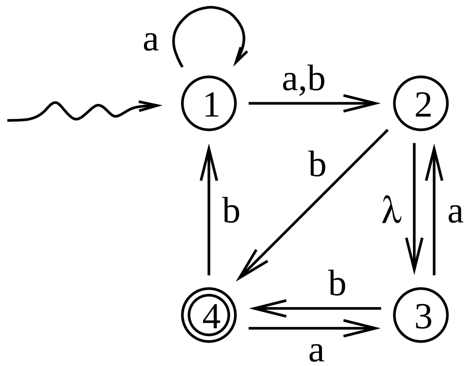
There are two states. The start state is 0 and 1 is a final state. There is a self-loop at state 0 labeled by the six triples: $z/a/az$, $z/b/bz$, $a/a/aa$, $a/b/\lambda$, $b/a/\lambda$, $b/b/bb$. There is an arc from state 0 to state 1 labeled by the triple $z/\lambda/z$.

4. Let L be the language of all regular expressions over $\{a, b\}$. Give a context-free grammar for L.

Let E be the start state. The productions are:

$E \to E + E|EE|E^*|(E)$
$E \to a|b|$"λ"$|$"$\emptyset$"

5. Consider the NFA shown below.



(i) Write a transition table for a minimal DFA equivalent to that NFA.

The start state is $\{1\}$. The final states are those sets which contain 4.

	a	b
$\{1\}$	$\{1,2,3\}$	$\{2,3\}$
$\{2,3\}$	$\{2,3\}$	$\{4\}$
$\{1,2,3\}$	$\{1,2,3\}$	$\{2,3,4\}$
$\{4\}$	$\{2,3\}$	$\{1\}$
$\{2,3,4\}$	$\{2,3\}$	$\{1,4\}$
$\{1,4\}$	$\{1,2,3\}$	$\{1,2,3\}$

(ii) Write a regular expression which describes the language accepted by that NFA.

$$a^*(a+b)(a^*b)(a(a+b)a^*b+aa^*b)^*$$

(iii) Give a regular grammar which generates the language accepted by that NFA.

The start symbol is S. The productions are:
$S \to aS|aA|bA$
$A \to B|bC$
$B \to aA|aB|bC$
$C \to aB|bS|\lambda$

However, a regular grammar is not allowed to have just a variable on the right hand side. Thus, we must delete $A \to B$ and insert two more productions:
$S \to aS|aA|bA|aB|bB$
$A \to aA|bC$
$B \to aA|aB|bC$
$C \to aB|bS|\lambda$