Computer Science 456/656 Spring 2021 Practice Examination April 8, 2021

The entire examination is 655 points. The actual exam will be shorter.

1. True or False. [5 points each] T = true, F = false, and O = open, meaning that the answer is not known to science at this time.

   i ________ Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, where $m, n \geq 0$. Then L is a regular language.

   ii ________ The complement of every regular language is regular.

   iii ________ The Kleene closure of every context-free language is context-free.

   iv ________ If a language has an ambiguous context-free grammar, then it is is not accepted by any deterministic push-down automaton.

   v ________ There is a PDA that accepts all valid C++ programs.

   vi ________ The intersection of any two regular languages is regular.

   vii ________ The intersection of any two context-free languages is context-free.

   viii ________ The set of all base 7 numerals for positive integers n such that $n \% 3 = 2$ is regular.

   ix ________ Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n c^m$, where $m, n \geq 0$. Then L is a context-free language.

   x ________ Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, where $m \geq n$. Then L is a context-free language.

   xi ________ The complement of every context-free language is context-free.

   xii ________ The union of any two context-free languages is context-free.

   xiii ________ If a language has an context-free grammar, then it is is accepted by some push-down automaton.

   xiv ________ Every context-free language has an unambiguous context-free grammar.

   xv ________ Every language that has an unambiguous context-free grammar is accepted by some DPDA.

   xvi ________ Every deterministic machine is a non-deterministic machine.

   xvii ________ The language consisting of all base 2 numerals for integer powers of 2 is regular.

   xviii ________ There is a DPDA that accepts the language of all palindromes over the binary alphabet $\{0, 1\}$.

   xix ______ The language $\{a^n b^n c^n d^n \mid n \geq 0\}$ is recursive.

   xx ______ The problem of whether a given context-free grammar generates all strings is decidable.

xxi ______ The language $\left\{a^i b^j c^k \mid j \geq i + k\right\}$ is context-free.

xxii ______ If a language L is undecidable, there is no machine that enumerates L in canonical order.

xxiii If $\mathcal{L}$ is a class of languages, co-$\mathcal{L}$ is defined to be the class of all languages that are not in $\mathcal{L}$.
______ Let $\mathcal{RE}$ be the class of all recursively enumerable languages. If L is in $\mathcal{RE}$ and also L is in co-$\mathcal{RE}$, then L must be decidable.

xxiv ______ The context-free grammar equivalence problem is in the class $\mathcal{RE}$.

xxv ______ The context-free grammar equivalence problem is in the class co-$\mathcal{RE}$.

xxvi ______ Every bounded function is recursive.

xxvii ______ If P is a mathematical proposition that can be stated using n binary bits, and P has a proof, then P must have a proof whose length is $O(2^{2^n})$.

xxviii ______ The complement of every recursive language is recursive.

xxix ______ The complement of every recursively enumerable language is recursively enumerable.

xxx ______ Every language which is generated by an general grammar is recursively enumerable.

xxxi ______ The context-free grammar equivalence problem is undecidable.

xxxii ______ Given any context-free grammar G and any string $w \in L(G)$, there is always a unique leftmost derivation of w using G.

xxxiii ______ For any deterministic finite automaton, there is always a unique minimal non-deterministic finite automaton equivalent to it.

xxxiv ______ Using multi-processors and other advanced technology, it is possible to design a machine which decides the halting problem.

xxxv ______ The question of whether two regular expressions are equivalent is $\mathcal{NP}$-complete.

xxxvi ______ The intersection of any context-free language with any regular language is context-free.

xxxvii ______ The halting problem is recursively enumerable.

xxxviii ______ The complement of every context-free language is context-free.

xxxix ______ No language which has an ambiguous context-free grammar can be accepted by a DPDA.

xl ______ The union of any two context-free languages is context-free.

xli ______ The question of whether a given Turing Machine halts with empty input is decidable.

xlii ______ The class of languages accepted by non-deterministic Turing Machines is the same as the class of languages accepted by Turing Machines.

xliii ______ Let $F(0) = 1$, and let $F(n) = 2^{F(n-1)}$ for $n > 0$. Then F is recursive.

xliv ________ Every language which is accepted by some non-deterministic machine is accepted by some deterministic machine.

xlv ________ The language of all regular expressions over the binary alphabet is a regular language.

xlvi ________ There is no computer program that decides whether any two C++ programs are equivalent.

xlvii ________ Every function that can be mathematically defined is recursive.

xlviii ________ The language of all binary numerals for multiples of 23 is regular.

xlix ________ The language of all binary strings which are the binary numerals for prime numbers is context-free.

l ________ Every bounded function from integers to integers is Turing-computable. (We say that f is *bounded* if there is some B such that $|f(n)| \leq B$ for all n.)

li ________ The language of all palindromes over $\{0, 1\}$ is inherently ambiguous.

lii ________ The language $\left\{a^i b^j c^k \;:\; i = j \text{ or } j = k\right\}$ is context-free, but is inherently ambiguous.

liii ________ Every context-free grammar can be parsed by some deterministic top-down parser.

liv ________ Every context-free grammar can be parsed by some non-deterministic top-down parser.

lv ________ Commercially available parsers cannot use the LALR technique, since most modern programming languages are not context-free.

lvi ________ The diagonal language is $\mathcal{RE}$.

lvii ________ The diagonal language is co-$\mathcal{RE}$.

lviii ________ The regular grammar membership problem is in $\mathcal{NC}$.

lix ________ The context-free grammar membership problem is in $\mathcal{NC}$.

lx ________ There is a polynomial time reduction of SAT to 3-CNF-SAT.

lxi ________ There is a language which is neither $\mathcal{RE}$ nor co-$\mathcal{RE}$.

lxii ________ $\mathcal{P}$-SPACE = co-$\mathcal{P}$-SPACE

lxiii ________ Regular expression equivalence is $\mathcal{P}$-SPACE complete.

2. Fill in the blanks.

(a) If there is an easy reduction from L_1 to L_2, then ________ is at least as hard as ________.

(b) If a language is accepted by some Turing machine, it is ____________________ enumerable.

(c) If L_1 is $\mathcal{NP}$ and L_2 is $\mathcal{NP}$-complete, there must be a $\mathcal{P}$-TIME reduction of ________ to ________.

(d) The language HALT is generated by an ________________________________ grammar.

3. [30 points] State the Church-Turing thesis, and explain (in about 5 lines or less) why it is important.

4. [30 points] We give an LALR parser for the context-free grammar with start symbol S and the given productions. (Think of b and n as begin and end, or as { and }.) Walk through a computation of that parser for the input string *bwwaaiaebnn*. The first configuration is $\$_0 : bwwaaiaebnn\$$.

1 $S \to a_2$

2 $S \to i_3 S_4$

3 $S \to i_3 S_4 e_5 S_6$

4 $S \to w_7 S_8$

5 $S \to b_9 L_{10} n_{11}$

6 $L \to L_{10} S_{12}$

7 $L \to \lambda$

	a	i	e	w	b	n	$	S	L
0	s2	s3		s7	s9			1	
1							**halt**		
2	r1	r1	r1	r1	r1	r1	r1		
3	s2	s3		s7	s9			4	
4	r2	r2	s5	r2	r2	r2	r2		
5	s2	s3		s7	s9			6	
6	r3	r3	r3	r3	r3	r3	r3		
7	s2	s3		s7	s9			8	
8	r4	r4	r4	r4	r4	r4	r4		
9	r7	r7		r7	r7	r7			10
10	s2	s3		s7	s9	s11		12	
11	r5	r5	r5	r5	r5	r5	r5		
12	r6	r6	r6	r6	r6	r6			

5. [5 points] What class of machines accepts the class of context free languages?

6. [5 points] What class of machines accepts the class of recursively enumerable languages?

7. [10 points] What is the *canonical order* of a language?

8. [10 points] What does it mean to say that machines M_1 and M_2 are *equivalent*?

9. [10 points] What does it mean to say that a language L is *decidable*?

10. [10 points] What is a *certificate* or *witness* as used in the definition of the class $\mathcal{NP}$?

11. [10 points] What does it mean to say that a language is in the class $\mathcal{P}$-SPACE?

12. [20 points] Find a Chomsky normal form grammar equivalent to the context-free grammar given below.

1 $S \to iS$

2 $S \to iSeS$

3 $S \to wS$

4 $S \to a$

13. [20 points] Let $\Sigma = \{0, 1\}$, the binary alphabet. We say a string w over Σ is *mostly positive* if $\#_1(w) > \#_0(w)$. Let L be the set of mostly positive strings over Σ.

Give a context-free grammar for L. **Very hard.**

14. [10 points] What is a **reduction** of a language L_1 to a language L_2?

15. (a) [10 points] State the pumping lemma for regular languages accurately. If you have all the right words but in the wrong order, that means you truly do not understand the lemma, and you might get no partial credit at all.

(b) [10 points] State the pumping lemma for context-free languages accurately. If you have all the right words but in the wrong order, that means you truly do not understand the lemma, and you might get no partial credit at all.

16. [20 points] Prove that a recursively enumerable language is accepted by some machine.

17. [20 points] Prove that a language is accepted by some machine if it is recursively enumerable.

18. [20 points] Prove that any language that can be recursively enumerated in canonoical order is recursive.

19. [20 points] Prove that a recursive language can be recursively enumerated in canonoical order.

20. [20 points] Give a polynomial time reduction of the subset sum problem to the partition problem.

21. [20 points] Give a polynomial time reduction of 3-CNF-SAT to the independent set problem. You may use figures in your explanation.