CSC 456/656 Spring 2022 Final Examination 8:00-10:00 May 11, 2022

1. True/False/Open.

2. Alphabets, strings, languages.

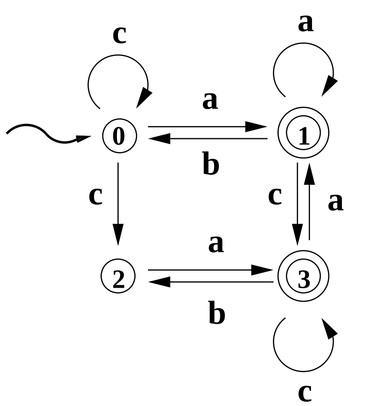
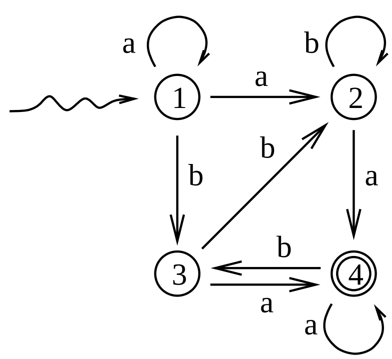
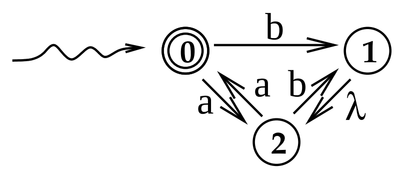
3. Give an NFA for a regular language.

   (i) Find an NFA with at most 4 states which accepts the language of binary strings which contain the substring 111.

   (ii) Let L be the language of all binary numerals for positive multiples of either 3 or 4, where leading zeros are not permitted. That is, $L = \{11, 100, 110, 1000, 1001, 1100, \ldots\}$. Find an NFA with 8 states which accepts L. There is also a DFA with 12 states which accepts L.

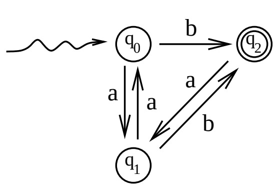
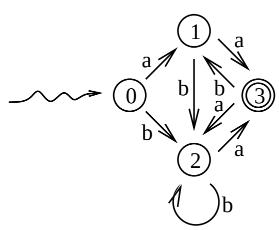
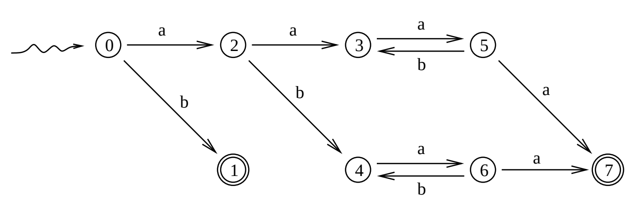
4. Convert an NFA into an equivalent DFA.

   Convert these NFAs to DFAs:

5. Minimize a DFA.

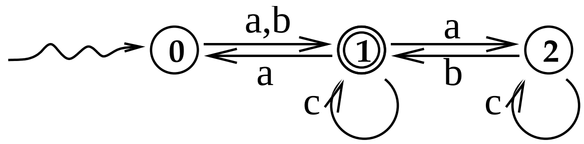
   Minimize these DFAs:

6. Equivalence of Regular Grammars and NFAs.
   Find an NFA which accepts the language generated by this regular grammar.

$$
\begin{aligned}
S &\to aA|cS|cC \\
A &\to aA|bS|cB|\lambda \\
B &\to aA|cB|bC|\lambda \\
C &\to aB
\end{aligned}
$$

7. Find a regular expression which describes the language accepted by an NFA.



8. Give a PDA for a context-free language.

   (i) Find a PDA for the language of all odd length palindromes over $\{a, b\}$.

   (ii) Find a DPDA for the language $\{a^i b^j c^k \,:\, j = i + k\}$

9. Give a context-free grammar for a context-free language. Let L be the language of all algebraic expressions where The operations are addition, subtraction and multiplication, there are parentheses, and the only variables are x, y, and z. Give an unambiguous CF grammar for L, where addition and subtraction have equal precedence, multiplication has precedence over addition and subtraction, and all three operations are left associative.

10. Chomsky normal form and the CYK algorithm.

Use the CYK algorithm to prove	$E \to E + T \mid E - T \mid T$
that $x + (y * (z - x) - z$ is not	$T \to T * F \mid F$
generated by the CF grammar:	$F \to (E) \mid x \mid y \mid z$

Hint: You must first find an equivalent CNF grammar.

11. Grammar classes: regular, context-free, context-sensitive, unrestricted.

   (i) Find a context-sensitive grammar for $\{a^n b^{2n} c^{3n} \,:\, n \geq 1\}$

   (ii) Is there an grammar which generates the language of all binary numerals for primes? What grammar class(es) does it belong to?

12. Parse trees and derivations.

   (i) Let L_1 be the language generated by the CF grammar given in 10. Give a left-most and a rightmost derivation of the string $x + y * z * (x - y)$.

13. Ambiguous and unambiguous CF grammars. Inherently ambiguous CF languages.

   Let $L_1 = \{a^i b^i c^j \,:\, i, j \geq 0\}$. Let $L_2 = \{a^i b^j c^j \,:\, i, j \geq 0\}$. Let $L = L_1 + L_2$.

   (i) Find a context-free grammar G for L.

   (ii) Is there an unambiguous context-free grammar for L?

   (iii) Prove that G is ambiguous by giving two different parse trees for some string.

14. LALR parsing.

   Design an LALR parser for the grammar

   1. $E \to E \, -_2 \, E_3$
   2. $E \to E \, *_4 \, E_5$
   3. $E \to x_6$

15. Complexity classes: $\mathcal{NC}$, $\mathcal{P}$–complete, $\mathcal{P}$–TIME, $\mathcal{NP}$, co–$\mathcal{NP}$, $\mathcal{NP}$–complete, $\mathcal{P}$–SPACE, recursive, recursively enumerable, co-RE, undecidable.

16. Closure of language classes under various operations.

    Which of the above complexity classes are closed under intersection?

17. Machine classes and language classes.

    For each class of machines, what is the class of languages it accepts?

18. Enumeration, enumeration in canonical order.

    (i) A language L is RE if and only if there is a machine that enumerates L.

    (ii) A language L is decidable if and only if there is a machine that enumerates L in canonical order.

    (iii) What does it mean to say that M enumerates L?

    (iv) What does it mean to say that M enumerates L in canonical order?

19. Both pumping lemmas.

    (i) State the pumping lemma for regular languages.

    (ii) State the pumping lemma for context-free languages.

20. Reduction definition of $\mathcal{NP}$.

    Fill in the blanks: L is $\mathcal{NP}$ if and only if there is a ________________________________ reduction of ________ to ________, where ________ is $\mathcal{NP}$–complete.

21. Certificate/verifier definition of $\mathcal{NP}$.

    Fill in the blanks:

    Given a language L, if there is a machine V_L such that, for every $w \in L$ there is a string c such that

    (i) $|c|$________________________________

    (ii) Given inputs ________ and ________ V_L halts in ________________________________

    then L is $\mathcal{NP}$.

22. Guide strings.

    What is a guide string? (Think about Theseus, Ariadne, and the Minotaur.)

23. $\mathcal{NP}$–complete problems.

    List as many $\mathcal{NP}$–complete problems, or languages, that we've studied this semester as you can.

24. Finding new $\mathcal{NP}$–complete problems using reduction.

    You could prove that L is $\mathcal{NP}$–complete as follows:

    (i) Show that L is ________________________________.

    (ii) Give a ________________________________ reduction of 3-SAT to L.

25. Give a polynomial time reduction of 3-SAT to the independent set problem.

26. Give a polynomial time reduction of the subset sum problem to the partition problem.

27. $\mathcal{NC}$ problems.

    (i) List a number of well-known polynomial time problem that are now known to be $\mathcal{LC}$.

    (ii) What is the importance nowadays of $\mathcal{NC}$?

28. $\mathcal{P}$–completeness.

    Name a $\mathcal{P}$–complete problem.

29. Problems known or believed to be harder than $\mathcal{NP}$.

    What complexity classes contain the following problems?

    (i) Sliding block problems.

    (ii) Games such Chess, Checkers, and Go: the problem is, given a game configuration, is it true that ______________________________?

30. Undecidable problems.

31. Prove that the halting problem is undecidable.

32. Turing Machines.

    (i) What is a Turing machine?

    (ii) Give a TM that shifts its input one space to the right.

33. Church-Turing thesis.

    Why is it important?

34. Countable and uncountable sets.

    What does it mean for a set to be countable, also known as enumerable, or denumerable? Which of the following sets are countable? (a) Integers (b) Rationals (c) Real numbers (d) The set of all binary strings (e) The set of all binary languages (f) The set of recursive binary languages (g) The set of RE binary languages (h) The set of co-RE binary languages (i) The set of undecidable binary languages (i) The set of recursive functions from binary strings to binary strings (j) The set of all functions from binary strings to binary strings.

35. Uncomputable functions, also called non-recursive functions.

    Describe one non-recursive function.

36. Recursive real numbers. https://www.ams.org/journals/proc/1954-005-05/ S0002-9939-1954-0063328-5/S0002-9939-1954-0063328-5.pdf

    What does it mean to say that a real number is recursive? Is the set of recursive real numbers countable?

37. Truth vs Reason.

    Not all truth can be discovered by reason: prove that there is some mathematical statement that is true but which has no mathematical proof.