Assignment 5: Due Saturday April 1, 2023, 23:59

There is a change of the schedule. Due to the delayed due date, more problems will be added to this assignment later.

Name:__________________________________________________________

You are permitted to work in groups, get help from others, read books, and use the internet.

1. True or False. T = true, F = false, and O = open, meaning that the answer is not known science at this time.
   
   (i) ______ Every $\mathcal{NP}$ language is decidable.
   
   (ii) ______ Every sliding block problem is $\mathcal{NP}$-complete.
   
   (iii) ______ Every undecidable language is $\mathcal{NP}$-complete.
   
   (iv) ______ If the Boolean circuit problem is $\mathcal{NC}$, then $\mathcal{P} = \mathcal{NC}$.
   
   (v) ______ If $L$ is any $\mathcal{NP}$ language, and if $w \in L$, there must be a polynomial time proof that $w \in L$.
   
   (vi) ______ Some sliding block problems are $\mathcal{P}$-space complete.

2. For each of these problems, or languages, which ones are known to be $\mathcal{NP}$-complete? Write Yes if it is known to be $\mathcal{NP}$-complete, No otherwise. You may need to do some research. Note: Open is not an answer.
   
   (i) ______ The firehouse problem. Given a graph $G$ and numbers $k$ and $d$, does there exist a set $F$ of $k$ vertices of $G$ such that every vertex of $G$ can be can be reached from some member of $F$ by a path of length at most $d$?
   
   (ii) ______ 3-SAT.
   
   (iii) ______ 2-SAT.
   
   (iv) ______ 4-SAT.
   
   (v) ______ Block sorting. Given a list, a block move consists of moving some contiguous sublist to another position. For example, the list AZCMXOPDYFQYS can be changed to AZCDYFMXOPQYS in one block move, by moving the sublist MXOP. The problem is, given a list $\ell$ and a number $k$, can $\ell$ be sorted in at most $k$ block moves?
   
   (vi) ______ The minimum spanning tree problem for weighted graphs.
   
   (vii) ______ The limited degree minimum spanning tree problem. The same as the previous problem, except that the degree of the spanning tree cannot exceed a given number.
   
   (viii) ______ Determining whether there is a solution for a given configuration of Rush Hour.
   
   (ix) ______ Given a set of items of various sizes and a set of trucks of various capacities, can the items be loaded into the trucks?
   
   (x) ______ Given binary numerals for integers $n$ and $a$, does there exist an integer $d$ that divides $n$ and is also between 2 and $a$?
3. Give a Chomsky Normal Form grammar equivalent to the grammar given below, where $E$ is the start symbol.

$E \rightarrow E + E$
$E \rightarrow E - E$
$E \rightarrow \neg E$
$E \rightarrow x$
$E \rightarrow y$

4. Consider the Chomsky Normal Form grammar $G$ given below, where $S$ is the start symbol.

$S \rightarrow IS$
$S \rightarrow WS$
$S \rightarrow XY$
$X \rightarrow IS$
$Y \rightarrow ES$
$S \rightarrow a$
$I \rightarrow i$
$W \rightarrow w$
$E \rightarrow e$

(a) Show that $G$ is ambiguous by giving two different leftmost derivations for the string $iiaea$.

(b) Use the CYK algorithm to prove that $iwiaeewwa \in L(G)$.
5. Give a polynomial time reduction of 3SAT to the independent set problem.

6. Give a polynomial time reduction of the subset sum problem to partition.
7. State the Church-Turing thesis. Why is it important?