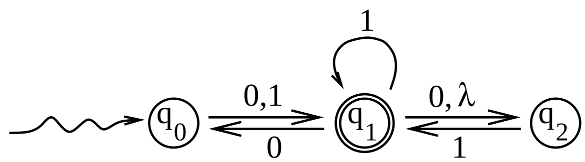
University of Nevada, Las Vegas Computer Science 456/656 Spring 2021

Answers to Assignment 1: Due Wednesday January 25, 2023

1. Which of the strings 00, 01001, 10010, 000, 0000 are accepted by the following NFA?



The computation with each input is shown. At each step, the state of the equivalent DFA, which is a member of 2^Q, is shown. The '$*$' indicates that the state is final.

	0	1
0	12	12
$*1$	02	12
2	$\emptyset$	12
02	12	12
$*12$	02	12

(a) Computation of 00:
$0 \xrightarrow{0} 12 \xrightarrow{0} 02$
Not accepted.

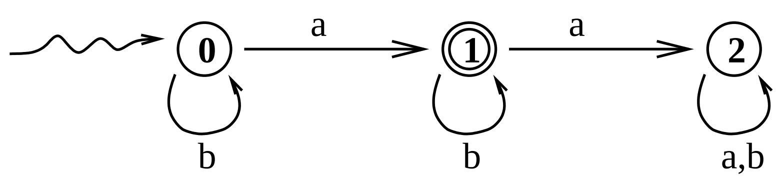
(b) Computation of 01001:
$0 \xrightarrow{0} 12 \xrightarrow{1} 12 \xrightarrow{0} 02 \xrightarrow{0} 12 \xrightarrow{1}, 12$
Accepted.

(c) Computation of 10010:00
$0 \xrightarrow{1} 12 \xrightarrow{0} 02 \xrightarrow{0} 12 \xrightarrow{1} 12 \xrightarrow{0}, 02$
Not accepted.

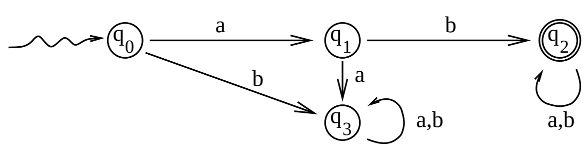
(d) Computation of 000:
$0 \xrightarrow{0} 12 \xrightarrow{0} 02 \xrightarrow{0} 12$
Accepted.

(e) Computation of 0000:
$0 \xrightarrow{0} 12 \xrightarrow{0} 02 \xrightarrow{0} 12$
$\to 0\,01$
Not accepted.

2. Construct a DFA which accepts the language $\{b^i a b^j : i, j \geq 0\}$, the language of all strings over $\{a, b\}$ which contain exactly one a.

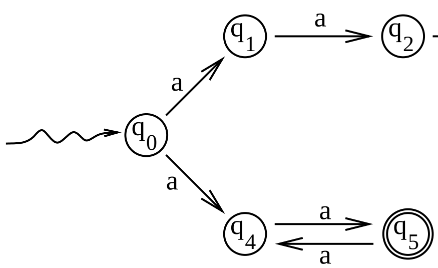


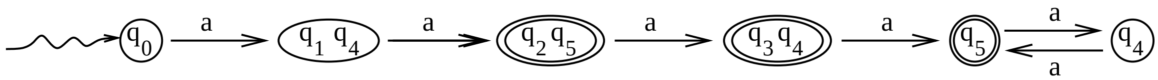
3. Can you find a DFA with three states that accepts the language of the figure given below? If not, can you give convincing arguments that no such DFA can exist?



Impossible. $L(q_0)$, $L(q_1)$, $L(q_2)$ and $L(q_3)$ are λ, a, $ab(a+b)^*$, and $aa(a+b)^*$, respectively. These regular expressions describe differerent languages, so no two states are equivalent.

4. Find a DFA equivalent to the NFA shown below.





5. Answer the 12 questions on page 3 of the handout *regular-I.pdf.*

   (a) Since $|L_1| = |L_2| = 3$, we would expect that $|L_1 L_2| = 9$. But it's only 8. Why?

   Because the string *babb* is obtained in two different ways.

   (b) Recall that $\emptyset$ is the empty language. If L is some language, what is the concatenation $\emptyset L$?

   $\emptyset$. The concatenation of the empty language with any language is the empty language.

   (c) Let $L_1 = \{\lambda\}$. the language consisting of only the empty string. If L_2 is some other language, what is the concatenation $L_1 L_2$?

   L_2. The concatenation of $\{\lambda\}$ with any language is that language.

(d) Given two languages L_1 and L_2, is the equation $L_1 L_2 = L_2 L_1$ always true?

No, but it's not always false, either.

(e) What is L^0?

No matter what L is, $L^0 = \{\lambda\}$.

(f) Is the equation $L_1(L_2 + L_3) = L_1 L_2 + L_1 L_3)$ always true?

Yes. That is one of the associative laws.

(g) What is $\emptyset^*$, the Kleene closure of the empty language?

$\{\lambda\}$. For any language L, $\lambda \in L^*$.

(h) What is L^{**}?

Kleene closure is idempotent, that is, $L^* * = L^*$.

(i) Is the union of two regular languages always regular?

Yes.

(j) Is the intersection of two regular languages always regular?

Yes.

(k) Is the complement of a regular language always regular?

Yes.

(l) Is the Kleene closure of a regular language always regular?

Yes.

Union and intersection are idempotent, that is, $L + L = L$ and $L \cap L = L$.

6. Solve problems given in the handout `finiteAutomata.pdf` associated with the following figures.

(a) Figure 1.

L be the language accepted by the DFA. $L = \{10, 101, 1000, 1011, 1110, \ldots\}$ The binary numeral for n is in L if and only if n is positive and equivalent to 2 modulo 3, that is, if $n\%3 = 2$.

(b) Figure 3.

The original figure is lacking a dead state. In my class, I allow you to not draw the dead state of a DFA. If it were drawn, we would call it state 8. We find that states 3 and 4 are equivalent, as are states 5 and 6, as are states 1 and 7. There is still a dead state. Here is the transition table of the minimal DFA.

	a	b
0	2	1
1/7	8	8
2	3/4	3/4
3/4	5/6	8
5/6	1/7	3/4
8	8	8

(c) Figure 5.

There are infinitely many correct answers. My answer is $a^*b(b + a(a^*(b + c))^*$, or alternatively, $a^*b(b + aa^*b + aa^*c)^*$

(d) Figure 11.

There are infinitely many correct answers. Here the obvious one:
$(aa + ab^*a + bb^*a)^*$
This can be simplified to: $(aa + (a + b)b^*a)^*$.
This can be simplified to:
$(a + b)b^*a)^*$
That may be the simplest answer, but I'm not sure.