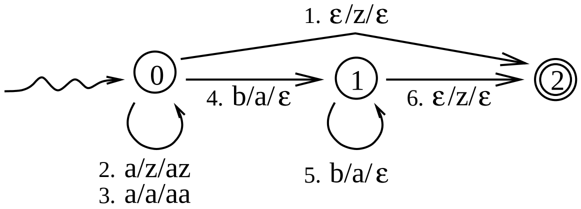
University of Nevada, Las Vegas Computer Science 456/656 Spring 2023

Answers to Assignment 3: Due Saturday February 25, 2023

1. Read the handout `pdaDef.pdf` on my website or on canvas.

   (a) The PDA M_1 given as Example 1 accepts the language. $L = \{a^n b^n \,:\, n \geq 0\}$.



   (b) Is M_1 a DPDA? Explain your answer.

   No. At the very first step, M will pop z. It does not know whether the input string is empty, so it does not know what to do. If the input is empty, it should execute transition 1 and halt, otherwise it should execute transition 2; read a and push az.

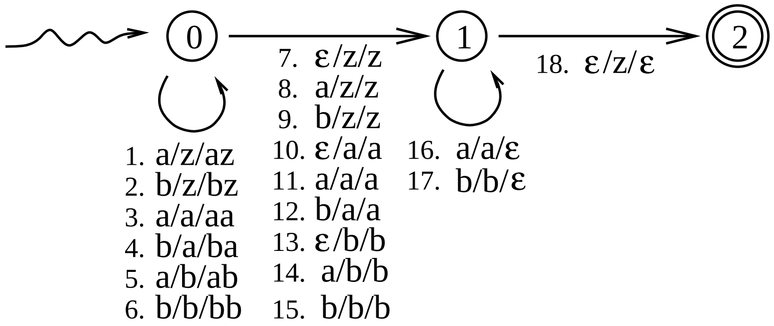
2. The PDA M_2 given as Example 2 in that handout accepts the language L of all palindromes over $\{a, b\}$.

   (a) Using the transitions given, write an accepting computation of M_2 for the input string *abbabba*.

   q_2 is the final state.

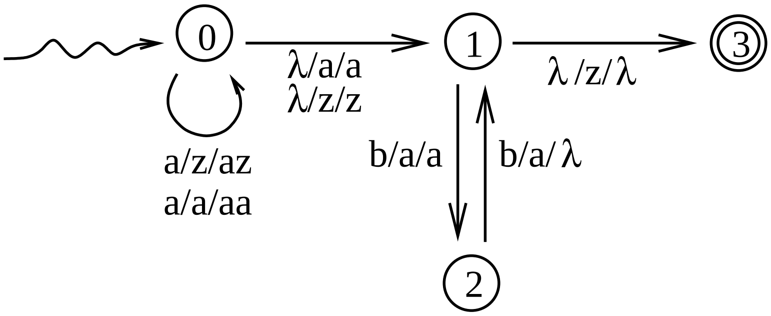
$$(q_0, abbabba, z) \overset{1}{\vdash}_M (q_0, bbabba, az) \overset{4}{\vdash}_M (q_0, babba, baz) \overset{6}{\vdash}_M (q_0, abba, bbaz) \overset{14}{\vdash}_M (q_1, bba, bbaz)$$
$$\overset{17}{\vdash}_M (q_1, ba, baz) \overset{17}{\vdash}_M (q_1, a, az) \overset{16}{\vdash}_M (q_1, \varepsilon, z) \overset{18}{\vdash}_M (q_2, \varepsilon, \varepsilon)$$

   (b) Draw a diagram of M_2.



3. What language does th PDA illustrated below accept? Hint: Instead of just staring at it, hoping for inspiration, try some strings; such as *aaaabbbb*.

   Answer: $\{a^n b^{2n} \,:\, n \geq 0\}$ Do you see how it works?



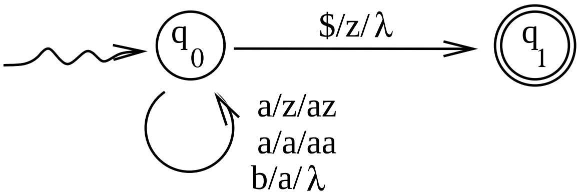
4. The Dyck language, strings where left and right parentheses match in the usual way. For example, $()(()) \in L_{\text{DYCK}}$.

   Since left and right parentheses look similar, expecially if you write them in a hurry, we substitute a and b for left and right parentheses. Thus, $abaabb \in L_{\text{DYCK}}$. A more formal definition is that L_{DYCK} is the set of all strings over $\{a, b\}$ which have equal numbers of a's and b's, and every prefix of which has at least as many a's as b's.

   Design a DPDA which accepts L_{DYCK}.

   As is the usual custom when designing a DPDA, we assume that the machine can detect the end of the input file. For that reason, we assume that there is an end-of-file symbol, which we call \$.

   In my class this semester, we assume that all PDA accept by empty stack, final state, and empty input file; all three conditions must hold for a PDA to accept the input string. This rule includes DPDA.



5. True or False. T = true, F = false, and O = open, meaning that the answer is not known to science at this time. You may need to search the handouts, or even the internet, for answers to some of these questions.
   Recall that "enumerable" and "countable" have the same meaning.

   (i) **T** The set of integers is countable.

   (ii) **T** The set of prime integers is countable.

   (iii) **T** The set of rational numbers is countable.

   (iv) **F** If a language L is countable, there must be machine which enumerates L.

   (v) **F** The set of real numbers is countable.

(vi) **T** Every language is countable.

(vii) **F** The set of all languages over the binary alphabet is countable.

(viii) **T** The set of all decidable languages over the binary alphabet is countable.

(ix) **T** The set of recursively enumerable languages over the binary alphabet is countable.

(x) **T** The intersection of any two decidable languages is decidable.

(xi) **T** The complement of any undecidable language is undecidable.

(xii) **T** The complement of any decidable language is decidable.

(xiii) **T** The halting problem is recursively enumerable.

(xiv) **F** The context-free grammar equivalence problem is recursively enumerable.

(xv) **T** Every subset of an enumerable set is enumerable.

(xvi) **F** Every subset of a recursively enumerable set is recursively enumerable.

(xvii) **T** There is a mathematical statement which is true but has no logical proof. (This does not mean, "No proof has been found." It means that no proof could **ever** be found.)

6. State the pumping lemma for context-free languages.

This is written in the handout **pumping.pdf**. It will appear on an examination, and the usual rules apply; if you have all the right words but your answer doesn't use the correct logic, you could (possibly) get no partial credit.