University of Nevada, Las Vegas Computer Science 456/656 Spring 2023

Review 1

1. True, False, or Open.

   (a) **T** If S is an infinite set, then 2^S must be uncountable.

   (b) **T** All standard arithmetic and matrix operations, as well as square root, are $\mathcal{NC}$.

2. Label each of the following sets as countable or uncountable.
   countable The set of integers.
   uncountable The set of real numbers.
   countable The set of rational real numbers.
   uncountable The set of irrational real numbers.
   uncountable The set of binary languages.
   countable The set of co-RE binary languages.
   countable The set of decidable binary languages.
   uncountable The set of undecidable binary languages.
   uncountable The set of unary languages.
   uncountable The set of functions from integers to integers.
   countable The set of recursive real numbers.
   countable The set of algebraic numbers.
   A number is algebraic if it is a root of a polynomial with integral coefficients.

3. Each language class is closed under which operators? Write “T.” “F,” or ”O” in each cell.

	union	intersection	complement	concatenation	Kleene closure
regular	T	T	T	T	T
$\mathcal{NC}$	T	T	T	T	T
context-free	T	F	F	T	T
$\mathcal{P}$–TIME	T	T	T	T	T
$\mathcal{NP}$	T	T	O	T	T
co-$\mathcal{NP}$	T	T	O	T	T
$\mathcal{P}$–SPACE	T	T	T	T	T
context-sensitive	T	T	T	T	T
recursive (decidable)	T	T	T	T	T
recursively enumerable	T	T	F	T	T
co-recursively enumerable	T	T	F	T	T
undecidable	F	F	T	F	F

4. Which of these problems, or languages, are **known** to be $\mathcal{NP}$–complete? (Write T or F)
   T TSP (traveling salesman)
   T partition
   T block sorting
   F equivalence of DFAs
   F equivalence of NFAs
   F equivalence of regular expressions
   F equivalence of regular grammars
   F equivalence of context-free grammars
   F Boolean circuit problem
   F 2SAT
   T 3SAT
   T 4SAT
   F generalized checkers (any size board)
   T vertex cover
   T independent set
   T dominating set
   F integer factoring with binary numerals
   F Rush Hour
   F Hex (the game)
   F Nim (the game)

5. Fill in the ACTION and GOTO tables of an LALR parser for the grammar given below, with start symbol E.

1. $E \to E +_2 E_3$
2. $E \to E -_4 E_5$
3. $E \to -_6 E_7$
4. $E \to E *_8 E_9$
5. $E \to (_{10} E_{11})_{12}$
6. $E \to x_{13}$

	x	$+$	$-$	$*$	(	)	\$	E
0	$s13$		$s6$		$s10$			1
1		$s2$	$s4$	$s8$			**halt**	
2	$s13$		$s6$		$s10$			3
3		$r1$	$r1$	$s8$		$r1$	$r1$	
4	$s13$		$s6$		$s10$			5
5		$r2$	$r2$	$s8$		$r2$	$r2$	
6	$s13$		$s6$		$s10$			7
7		$r3$	$r3$	$r3$		$r3$	$r3$	
8	$s13$		$s6$		$s10$			9
9		$r4$	$r4$	$r4$		$r4$	$r4$	
10	$s13$		$s6$		$s10$			11
11		$s2$	$s4$	$s8$		$s12$		
12		$r5$	$r5$	$r5$		$r5$	$r5$	
13		$r6$	$r6$	$r6$		$r6$	$r6$	

6. Give a proof that the set of real numbers $\mathbb{R}$

   Proof: By contradiction. Assume that $\mathbb{R}$ is countable, that is, $\mathbb{R}$ has an enumeration $x_1, x_2, \ldots$. For each $i \geq 1$, let d_i be the digit in the 10^{-i} place of the decimal expansion of x_i. Let x be the real number whose decimal expansion has the digit d_i' in the 10^{-i} place, where $d_i' = \begin{cases} 1 \text{if} d_i = 0 \\ 0 \text{otherwise} \end{cases}$ Then $x \in \mathbb{R}$ hence must equal x_i for some i. But $x \neq x_i$ since $d_i \neq d_i'$, contradiction. Thus, $\mathbb{R}$ is not countable.