University of Nevada, Las Vegas Computer Science 456/656 Spring 2021

Practice Problems for the Examination on April 12, 2023

1. Review answers to homework5:
   `http://web.cs.unlv.edu/larmore/Courses/CSC456/S23/Assignments/hw5ans.pdf`

2. Review answers to homework6:
   `http://web.cs.unlv.edu/larmore/Courses/CSC456/S23/Assignments/hw6ans.pdf`

3. True or False. If the question is currently open, write "O" or "Open."

   (i) ________ The language of all binary strings which are the binary numerals for multiples of 23 is regular.

   (ii) ________ If L is an $\mathcal{RE}$ (recursively enumerable) language and $w \notin L$, there must be proof that $w \notin L$.

   (iii) ________ If L is a co-$\mathcal{RE}$ language and $w \notin L$, there must be proof that $w \notin L$.

   (iv) ________ If L is a $\mathcal{P}$–SPACE language and $w \in L$, there must be a proof of polynomial length that $w \in L$.

   (v) ________ If L is any $\mathcal{P}$–TIME language, there is a reduction of L to the Boolean circuit problem, and this reduction can be calculated in polylogarithmic time with polynomially many processors.

   (vi) ________ Let $L = \{\langle G_1\rangle\langle G_2\rangle : G_1$ is not equivalent to $G_2\}$ Then L is recursively enumerable.

   (vii) ________ The complement of any $\mathcal{P}$-SPACE language is $\mathcal{P}$-SPACE.

   (viii) ________ The complement of any $\mathcal{NP}$ language is $\mathcal{NP}$.

   (ix) ________ The complement of every recursive language is recursive.

   (x) ________ The complement of every recursively enumerable language is recursively enumerable.

   (xi) ________ Every language which is generated by a general grammar is recursively enumerable.

   (xii) ________ The context-free language membership problem is undecidable.

   (xiii) ________ The factoring problem, where inputs are written in binary notation, is co-$\mathcal{NP}$.

   (xiv) ________ The factoring problem, where inputs are written in unary (caveman) notation, is $\mathcal{P}$–TIME.

   (xv) ________ For any non-deterministic finite automaton, there is always a unique minimal deterministic finite automaton equivalent to it.

   (xvi) ________ The question of whether two regular expressions are equivalent is known to be $\mathcal{NP}$-complete.

   (xvii) ________ The halting problem is recursively enumerable.

   (xviii) ________ The intersection of any two context-free languages is context-free.

   (xix) ________ The question of whether a given Turing Machine halts with empty input is decidable.

(xx) ________ The class of languages accepted by NTM's (non-deterministic Turing machines) is the same as the class of languages accepted by Turing machines.

(xxi) ________ The class of languages accepted by non-deterministic push-down automata is the same as the class of languages accepted by deterministic push-down automata.

(xxii) ________ Let π be the ratio of the circumference of a circle to its diameter. The problem of whether the n^{th} digit of the decimal expansion of π for a given n is equal to a given digit is decidable.

(xxiii) ________ An undecidable language is necessarily $\mathcal{NP}$-complete.

(xxiv) ________ Every context-free language is in the class $\mathcal{P}$-TIME.

(xxv) ________ Every regular language is in the class $\mathcal{NC}$

(xxvi) ________ Let $L = \{a^i b^j c^k \,:\, i = j \text{or} j = k\}$. Then L is not generated by any unambiguous context-free grammar.

(xxvii) ________ Every context-free grammar can be parsed by some deterministic top-down parser.

(xxviii) ________ Every context-free grammar can be parsed by some non-deterministic top-down parser.

(xxix) ________ Commercially available parsers do not use the LALR technique, since most modern programming languages are not context-free.

(xxx) ________ The boolean satisfiability problem is undecidable.

(xxxi) ________ If anyone ever proves that the binary integer factorization problem is in $\mathcal{P}$–TIME, then all public key/private key encryption systems will be known to be insecure.

(xxxii) ________ If anyone ever proves that $\mathcal{P} = \mathcal{NP}$, then all public key/private key encryption systems will be known to be insecure.

(xxxiii) ________ If a string w is generated by a context-free grammer G, then w has a unique leftmost derivation if and only if it has a unique rightmost derivation.

(xxxiv) ________ A language L is in $\mathcal{NP}$ if and only if there is a polynomial time reduction of L to SAT.

(xxxv) ________ Every subset of a regular language is regular.

(xxxvi) ________ The intersection of any context-free language with any regular language is context-free.

(xxxvii) ________ Every language which is generated by a general grammar is recursively enumerable.

(xxxviii) ________ There exists some mathematical statement which is true but which has no proof.

(xxxix) ________ The set of all binary numerals for prime numbers is in the class $\mathcal{P}$.

(xl) ________ If L_1 reduces to L_2 in polynomial time, and if L_2 is $\mathcal{NP}$, and if L_1 is $\mathcal{NP}$-complete, then L_2 must be $\mathcal{NP}$-complete.

(xli) ________ Given any context-free grammar G and any string $w \in L(G)$, there is always a unique leftmost derivation of w using G.

(xlii) ________ For any deterministic finite automaton, there is always a unique minimal non-deterministic finite automaton equivalent to it.

(xliii) ________ No language which has an ambiguous context-free grammar can be accepted by a DPDA.

(xliv) ________ The class of languages accepted by non-deterministic push-down automata is the same as the class of languages accepted by deterministic push-down automata.

(xlv) ________ Let $F(0) = 1$, and let $F(n) = 2^{F(n-1)}$ for $n > 0$. Then F is recursive.

(xlvi) ________ The "Busy beaver" function is recursive.

(xlvii) ________ Let π be the ratio of the circumference of a circle to its diameter. (That's the usual meaning of π you learned in school.) The problem of whether the n^{th} digit of π, for a given n, is equal to a given digit is decidable.

(xlviii) ________ There is a machine that parses Pascal. (A parser for a computer language is a machine that constructs a correct parse tree for every valid program written in that language.)

(xlix) ________ There is a machine that parses C++. Hint: look this up on the internet.

(l) ________ Every function that can be mathematically defined is recursive.

(li) ________ Every context-free language is in the class $\mathcal{P}$-TIME.

(lii) ________ The Post correspondence problem is undecidable.

For the next three problems, recall that a fraction is a string consisting of a numeral, follwed by a slash, followed by another numeral. Thus, any set of fractions is a language. If x is any real number, let $L_L(x)$ be the set of all fractions whose values are less than x, and let $L_R(x)$ be the set of all fractions whose values are greater than x.

(liii) ________ If a sequence of fractions converges to a real number x, then x must be a recursive real number.

(liv) ________ If $L_L(x)$ is recursive, then then x must be a recursive real number.

(lv) ________ If $L_L(x)$ is recursively enumerable, then then x must be a recursive real number.

(lvi) ________ If $L_L(x)$ and $L_R(x)$ are both recursively enumerable, then x must be a recursive real number.

4. State a problem, or language, that is known to be in the class $\mathcal{NP}$, is not known to be $\mathcal{P}$–TIME, and is not known to be $\mathcal{NP}$–complete.

5. Determine whether the following 2CNF Boolean expression is satisfiable. If so, give a satisfying assignment.

   $(!d + g) * (!h+!d) * (f + e) * (e+!e) * (b+!j) * (!e + j) * (!i + c) * (a + d) * (g+!j) * (e+!c) * (!j + f)$
   $(b + i) * (d+!j) * (!h+!c) * (f + g) * (h+!c) * (!b+!j) * (!g+!j) * (a + c) * (!i + g)$

6. Prove that the halting problem is undecidable. Do it the way you should, not by quoting Lemma 2.