## CSC 456/656 Spring 2023 Answers to First Examination February

## 8, 2022

1. True or False. 5 points each. $\mathrm{T}=$ true, $\mathrm{F}=$ false, and $\mathrm{O}=$ open, meaning that the answer is not known science at this time.
(i) $\mathbf{F}$ Every subset of a regular language is regular.
(ii) $\mathbf{T}$ The class of regular languages is closed under intersection.
(iii) $\mathbf{O} \mathcal{P}$-TIME $=\mathcal{N} \mathcal{P}$.
(iv) $\mathbf{T}$ The class of regular languages is closed under Kleene closure.
(v) $\mathbf{T}$ The class of context-free languages is closed under union.
(vi) $\mathbf{F}$ The class of context-free languages is closed under intersection.
(vii) $\mathbf{F}$ The set of binary numerals for prime numbers is a regular language.
(viii) $\mathbf{T}$ The complement of any $\mathcal{P}$-Time language is $\mathcal{P}$-Time.
(ix) $\mathbf{F}$ The complement of any context-free language is context-free.
(x) $\mathbf{T}$ The complement of any recursive (that is, decidable) language is recursive.
(xi) $\mathbf{T}$ If $\Sigma$ is an alphabet, then $\Sigma^{*}$ is a regular language.
(xii) $\mathbf{F}$ If $L$ is a language and $L^{*}$ is a regular language, then $L$ must be a regular language. (Think!) Suppose $L=\left\{a^{2^{k}}: k \geq 0\right\}$, that is, $L=\{a, a a, a a a a, a a a a a a a a, \ldots\}$. Then $L$ is not regular, but $L^{*}=\left\{a^{n}: n \geq 0\right\}$ is regular.
(xiii) $\mathbf{F}$ The class of languges which are not regular is closed under intersection. (Think!)

Let $L_{1}=\left\{a^{n}: n\right.$ is prime $\}$, and Let $L_{2}=\left\{a^{n}: n+1\right.$ is prime $\}$. Neither of those languages is regular, but $L_{1} \cap L_{2}=\left\{a^{2}\right\}$ which is finite, hence regular.
(xiv) F A minimal DFA equivalent to an NFA with $n$ states must have $2^{n}$ states.

The minimal DFA cannot have more than $2^{n}$ states, but it could have fewer.
(xv) O If a non-derministic machine can solve a given problem in polynomial time, then there is a deterministic machine which can solve the same problem in polynomial time.
This statement is equivalent to $\mathcal{P}=\mathcal{N} \mathcal{P}$.
(xvi) $\mathbf{T}$ If a non-derministic machine can solve a given problem in polynomial time, then there is a deterministic machine which can solve the same problem in exponential time.
The problem can be solved by brute force: simply try everything.
2. [10 points] Give an example of a language which is context-free but not regular.

There are many examples. The simplest one is $\left\{a^{n} b^{n}: n \geq 0\right\}$
3. [10 points] Give an example of a language which is not context-free.

There are many examples. The simplest is $\left\{a^{n} b^{n} c^{n}: n \geq 0\right\}$. Some people gave a regular language. This is wrong, because every regular language is context-free.
4. [20 points] Let $L$ be the language of all binary strings encoding numbers which are equivalent to 1 modulo 3 , where leading zeros are allowed. Thus, $L=\{1,01,001,100,111,0100,0111,1010, \ldots\}$. Draw a DFA which accepts $L$. (You need only three states.)

5. [20 points] Let $G$ be the CF grammar given below, where $E$ is the start symbol. Show that $G$ is ambiguous by giving two different rightmost derivations for the string $x-y * z$.

1. $E \rightarrow E-E$
2. $E \rightarrow E * E$
3. $E \rightarrow x$
$E \Rightarrow E-E \Rightarrow E-E * E \Rightarrow E-E * z \Rightarrow E-y * z \Rightarrow x-y * z$
4. $E \rightarrow y$
$E \Rightarrow E * E \Rightarrow E * z \Rightarrow E-E * z \Rightarrow E-y * z \Rightarrow x-y * z$
5. $E \rightarrow z$
6. [20 points] Give a grammar for the language accepted by the NFA shown in Figure 1 below.


Figure 1: NFA for problems 6 and 9.

This grammar differs from what was given in class. I have eliminated the production that has one variable on the right-hand side.
$S \rightarrow a S|a A| b A|a B| b B$
$A \rightarrow a A|a B| b C$
$B \rightarrow a A|a B| b C$
$C \rightarrow a B|b S| \lambda$
7. [20 points] Give a regular expression for the language accepted by the following NFA


There are infinitely many correct answers. Here are two of the simplest.
$b(a+b)\left(a+b a^{*} b+b(a+b)\right)^{*}$
$(b a+b b)\left(a+b a^{*} b+b a\right)^{*}$
8. [20 points] State the pumping lemma for regular languages correctly. Pay close attention to the order in which you write the quantifiers. If you have all the correct words in the wrong order, you still might get no credit.

For any regular language $L$
there is a number $p$ such that
for any $w \in L$ of length at least $p$
there are strings $x, y$, and $z$ such that the following conditions hold:

1. $w=x y z$,
2. $|x y| \leq p$,
3. $y$ is not the empty string,
4. for any integer $i \geq 0, x y^{i} z \in L$.
5. [20 points] Draw a minimal DFA equivalent to the NFA shown in Figure 1 in problem 6 above. Show the transition table, and also show the matrix used for minimizing the DFA.

Of the 16 states of the DFA, only seven are reachable. States 3 and 23 are equivalent.

|  | a | b |
| ---: | :---: | :---: |
| 1 | 123 | 23 |
| 3 | 23 | ${ }^{*} 4$ |
| ${ }^{*} 4$ | 3 | 1 |
| ${ }^{*} 14$ | 123 | 123 |
| 23 | 23 | ${ }^{*} 4$ |
| 123 | 123 | ${ }^{*} 234$ |
| ${ }^{*} 234$ | 23 | ${ }^{*} 14$ |


|  | 1 | 3 | 4 | 14 | 23 | 123 | 234 |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | O | X | X | X | X | X | X |
| 3 | X | O | X | X | O | X | X |
| ${ }^{*} 4$ | X | X | O | X | X | X | X |
| ${ }^{*} 14$ | X | X | X | O | X | X | X |
| 23 | X | O | X | X | O | X | X |
| 123 | X | X | X | X | X | O | X |
| ${ }^{*} 234$ | X | X | X | X | X | X | O |



