University of Nevada, Las Vegas Computer Science 456/656 Spring 2024

Answers to Assignment 4: Due Saturday March 2, 2024

1. True or False, write T or F. If the answer is unknown to science at this time, write O, for Open.

   (a) **T** If L is both $\mathcal{RE}$ and co-$\mathcal{RE}$, then L is decidable.

   (b) **T** SAT is decidable.

   (c) **T** 2-SAT is $\mathcal{P}$–TIME.

   (d) **T** The binary numeral primality problem is $\mathcal{P}$–TIME.

   (e) **O** The binary numeral factorization problem is $\mathcal{P}$–TIME.

   (f) **T** $\mathcal{NP} \subseteq \mathcal{P}$–SPACE.

   (g) **T** Every context-sensitive language is decidable.

   (h) **T** The complement of every undecidable language is undecidable.

   (i) **T** The halting problem is $\mathcal{RE}$.

   (j) **F** The context-free grammar equivalence problem is $\mathcal{RE}$.

   (k) **F** Every undecidable language is either $\mathcal{RE}$ or co-$\mathcal{RE}$.

   (l) **F** There are countably many binary languages.

   (m) **T** Every language has a canonical order enumeration.

   (n) **F** For any real number x, the problem of whether a given rational number is less than x is decidable. (A rational number is a number that can be written as $\frac{p}{q}$, where p and q are integers.)

2. State the pumping lemma for context-free languages correctly.

   For any context-free language L
   there exists a number p such that
   for any $w \in L$ of length at least p
   there exist string u, v, x, y, and z such that
   the following four statements hold:
   1. $w = uvxyz$,
   2. $|vxy| \leq p$,
   3. $|v| + |y| \geq 0$,
   4. for any integer $i \geq 0$, $uv^ixy^iz \in L$.

3. Give a context-sensitive grammar for $L = \{a^n b^n c^n d^n \ : \ n \geq 1\}$

$\S \to abcd \,|\, aAbcd$
$Ab \to bA$
$Ac \to cA$
$Ad \to Bdd$
$cBd \to Bccd$
$cBc \to Bcc$
$bBc \to Bbbc$
$bBb \to Bbb$
$aB \to aa \,|\, aaA$

Generate strings $abcd$, $aabbccdd$ and $aaabbbcccddd$ using the grammar.

$\underline{S} \Rightarrow abcd$

$\underline{S} \Rightarrow a\underline{Ab}cdRightarrowab\underline{Ac}d \Rightarrow abc\underline{Ad} \Rightarrow ab\underline{cBd}d \Rightarrow a\underline{bBc}cdd \Rightarrow \underline{aB}bbccdd \Rightarrow aabbccdd$

$\underline{S} \Rightarrow a\underline{Ab}cd \Rightarrow ab\underline{Ac}d \Rightarrow abc\underline{Ad} \Rightarrow ab\underline{cBd}d \Rightarrow a\underline{bBc}cdd \Rightarrow \underline{aB}bbccdd \Rightarrow aa\underline{Ab}bccdd \Rightarrow aab\underline{Ab}ccdd \Rightarrow aabb\underline{Ac}cdd \Rightarrow aabbc\underline{Ac}dd \Rightarrow aabbcc\underline{Add} \Rightarrow aabbc\underline{cBd}ddd \Rightarrow aabbc\underline{Bc}cddd \Rightarrow aab\underline{bBc}cddd \Rightarrow aa\underline{bBb}bcccddd \Rightarrow a\underline{aB}bbbcccddd \Rightarrow aaabbbcccddd$

4. Prove that the halting problem is undecidable.

   See the handout.

5. Assuming that the subset sum problem is $\mathcal{NP}$–complete, prove that the partition problem is $\mathcal{NP}$–complete.

   See the handout.

6. Assuming thqt 3-SAT is $\mathcal{NP}$–complete, prove that the independent set problem is $\mathcal{NP}$–complete.

   See the handout.