CSC 456/656 Spring 2024 First Examination Problems to Study

1. True or False. T = true, F = false, and O = open, meaning that the answer is not known to science at this time.

   (i) _____ Every subset of a regular language is regular.

   (ii) _____ The class of regular languages is closed under intersection.

   (iii) _____ $\mathcal{P}$–TIME = $\mathcal{NP}$.

   (iv) _____ The class of regular languages is closed under Kleene closure.

   (v) _____ The class of context-free languages is closed under union.

   (vi) _____ The class of context-free languages is closed under intersection.

   (vii) _____ The set of binary numerals for prime numbers is a regular language.

   (viii) _____ The set of binary numerals for prime numbers is $\mathcal{P}$–TIME.

   (ix) _____ The complement of any $\mathcal{P}$-TIME language is $\mathcal{P}$-TIME.

   (x) _____ The complement of any context-free language is context-free.

   (xi) _____ The complement of any recursive (that is, decidable) language is recursive.

   (xii) _____ If Σ is an alphabet, then Σ^* is a regular language.

   (xiii) _____ If L is a language and L^* is a regular language, then L must be a regular language. **(Think!)**

   (xiv) _____ The class of languges which are **not regular** is closed under intersection. **(Think!)**

   (xv) _____ The minimal DFA equivalent to an NFA with n states must have at least 2^n states.

   (xvi) _____ If a non-derministic machine can solve a given problem in polynomial time, then there must be is a deterministic machine which can solve the same problem in polynomial time.

   (xvii) _____ The complement of any regular language is regular.

   (xviii) _____ Any context-free language is generated by some ambiguous context-free grammar.

   (xix) _____ Any context-free language is generated by some unambiguous context-free grammar.

   (xx) _____ The Dyck language is regular.

   (xxi) _____ Every regular language is context-free.

   (xxii) _____ Every language is decided by some machine.

   (xxiii) _____ Every language is accepted by some machine.

2. A language is context-free if and only if it is accepted by some _____.

3. Give definitions of each of the following terms.

   (a) Symbol.

   (b) Alphabet.

   (c) String over a given alphabet Σ.

   (d) Language over a given alphabet Σ.

4. Give an example of a language which is context-free but not regular.

5. Give an example of a language which is not context-free.

6. Let L be the language of all binary strings encoding numbers which are equivalent to 1 modulo 3, where leading zeros are allowed. Thus, $L = \{1, 01, 001, 100, 111, 0100, 0111, 1010, \ldots\}$. Draw a DFA which accepts L. (You need only three states.)

7. 20 Let G be the CF grammar given below, where E is the start symbol.

   (a) Show that G is ambiguous by giving two different **rightmost** derivations for the string $x-y+z$.

   (b) Which of these two derivations respects the usual precedence of operators?

   1. $E \to E - E$
   2. $E \to E + E$
   3. $E \to x$
   4. $E \to y$
   5. $E \to z$

8. Give a grammar for the language accepted by the NFA shown in Figure 1 below.

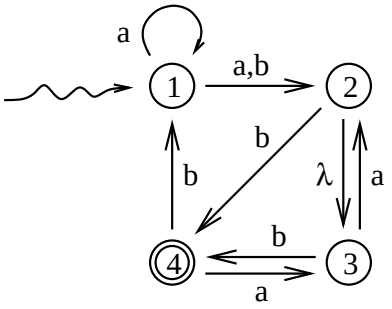

Figure 1:

NFA for Problem 8.

9. Write a regular expression for the language accepted by the machine shown in Figure 2.

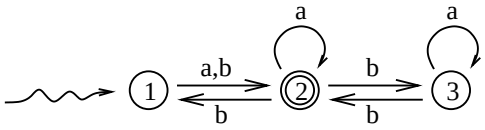

Figure 2

10. For each of the binary languages described here, identify which of grammars (each with start symbol S) listed below generates that language?

    (i) _____ All binary strings.

    (ii) _____ All binary numerals for multiples of three.

    (iii) _____ All binary strings which have the same number of 0's as 1's.

    (iv) _____ All binary strings w which have the Dyck property, that is, w has equal number of 0's and 1's, and each prefix of w has at least as many 1's as 0's.

    (v) _____ All binary numerals for powers of two.

    (vi) _____ The language accepted by the NFA shown in Figure 3 below.

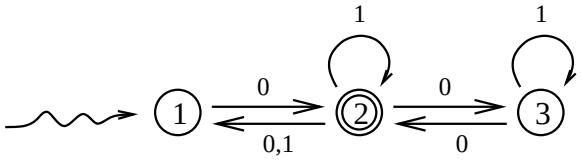

Figure 3

    (a) $S \to 1S0S \,|\, \lambda$

    (b) $S \to AS \,|\, BS \,|\, \lambda$
    $A \to 1A0A \,|\, \lambda$
    $A \to 0B1B \,|\, \lambda$

    (c) $S \to 1A$
    $A \to 0A \,|\, \lambda$

    (d) $S \to 1S \,|\, 0S \,|\, \lambda$

    (e) $S \to 1A \,|\, 0$
    $A \to 1B \,|\, 0C$
    $B \to 0B \,|\, 1A \,|\, \lambda$
    $C \to 1C \,|\, 0A$

    (f) $S \to 0A$
    $A \to 0S \,|\, 1S \,|\, 0B \,|\, 1A \,|\, \lambda$
    $B \to 1B \,|\, 0A$

11. Each of these regular expressions is for one of the languages given in Problem 10 above. Identify the correct language for each regular expression.

    A. _____ 10^*

    B. _____ $0(1 + (0 + 1)0 + 01^*0)^*$

    C. _____ $101^*01(0 + 01^*0 + 10 * 1)^*$

    D. _____ $(0 + 1)^*$

12. What are the four language (or grammar) classes of the Chomsky hierarchy? Be sure to mention the type numbers as well as the name of the class.

13. Draw a minimal DFA equivalent to the DFA shown in Figure 4. Show your work.

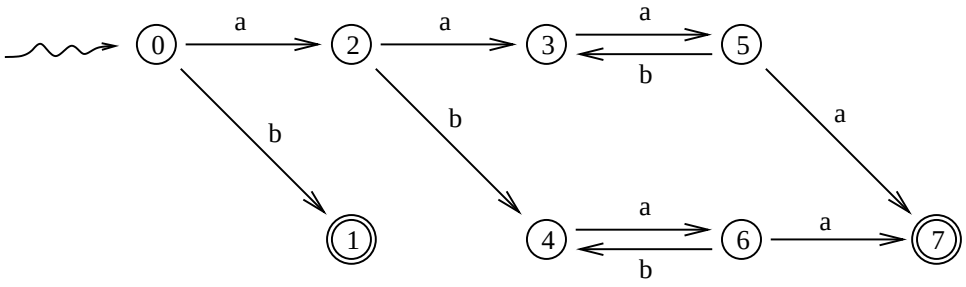

Figure 4

14. Draw a minimal DFA equivalent to the NFA shown in Figure 5. Show your work.

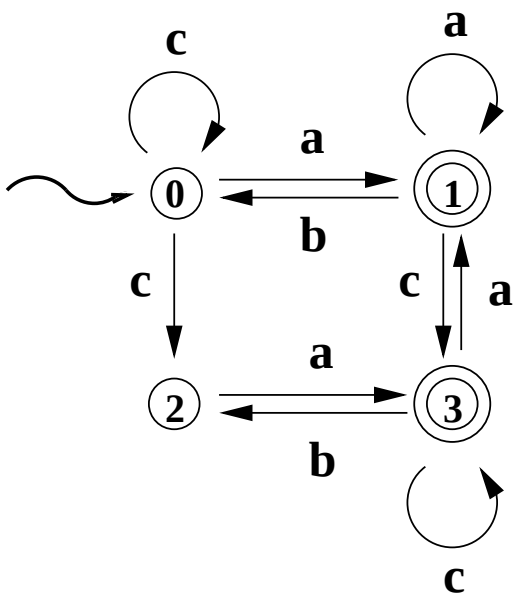

Figure 5