CSC 456/656 Spring 2024 First Examination Problems to Study

1. True or False. T = true, F = false, and O = open, meaning that the answer is not known to science at this time.

   (i) **F** Every subset of a regular language is regular.

   (ii) **T** The class of regular languages is closed under intersection.

   (iii) **O** $\mathcal{P}$–TIME $= \mathcal{NP}$.

   (iv) **T** The class of regular languages is closed under Kleene closure.

   (v) **T** The class of context-free languages is closed under union.

   (vi) **F** The class of context-free languages is closed under intersection.

   (vii) **F** The set of binary numerals for prime numbers is a regular language.

   (viii) **T** The set of binary numerals for prime numbers is $\mathcal{P}$–TIME.

   (ix) **T** The complement of any $\mathcal{P}$-TIME language is $\mathcal{P}$-TIME.

   (x) **F** The complement of any context-free language is context-free.

   (xi) **T** The complement of any recursive (that is, decidable) language is recursive.

   (xii) **T** If Σ is an alphabet, then Σ^* is a regular language.

   (xiii) **F** If L is a language and L^* is a regular language, then L must be a regular language. **(Think!)**

   (xiv) **T** The class of languges which are **not regular** is closed under intersection. **(Think!)**

   (xv) **F** The minimal DFA equivalent to an NFA with n states must have at least 2^n states.

   (xvi) **O** If a non-derministic machine can solve a given problem in polynomial time, then there must be is a deterministic machine which can solve the same problem in polynomial time.

   (xvii) **T** The complement of any regular language is regular.

   (xviii) Any context-free language is generated by some ambiguous context-free grammar. **T**

   (xix) Any context-free language is generated by some unambiguous context-free grammar. **F**

   (xx) **F** The Dyck language is regular.

   (xxi) **T** Every regular language is context-free.

   (xxii) **F** Every language is decided by some machine.

   (xxiii) **F** Every language is accepted by some machine.

2. A language is context-free if and only if it is accepted by some *push-down automaton*.

3. Give definitions of each of the following terms.

   (a) Symbol. There is no definition. Anything can be a symbol.

   (b) Alphabet. A finite set of symbols.

   (c) String over a given alphabet Σ. A finite sequence of members of Σ.

   (d) Language over a given alphabet Σ. A set of strings over Σ.

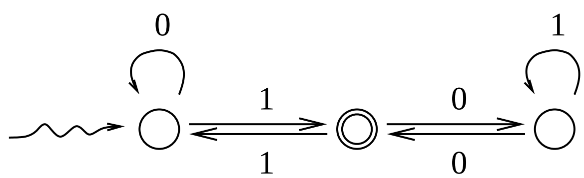
4. Give an example of a language which is context-free but not regular.

   $\{a^n b^n \,:\, n \geq 0\}$

5. Give an example of a language which is not context-free.

   $\{a^n b^n c^n \,:\, n \geq 0\}$

6. Let L be the language of all binary strings encoding numbers which are equivalent to 1 modulo 3, where leading zeros are allowed. Thus, $L = \{1, 01, 001, 100, 111, 0100, 0111, 1010, \ldots\}$. Draw a DFA which accepts L. (You need only three states.)



7. 20 Let G be the CF grammar given below, where E is the start symbol.

   (a) Show that G is ambiguous by giving two different **rightmost** derivations for the string $x - y + z$.

   (b) Which of these two derivations respects the usual precedence of operators? 1. $E \to E - E$
   2. $E \to E + E$
   3. $E \to x$
   4. $E \to y$
   5. $E \to z$

   $E \Rightarrow E - E \Rightarrow x - E \Rightarrow x - E * E \Rightarrow x - y * E \Rightarrow x - y * z$

   $E \Rightarrow E * E \Rightarrow E - E * E \Rightarrow x - E * E \Rightarrow x - y * E \Rightarrow x - y * z$

   The first derivation is the one that respects the usual precedence of the operators.

9. Write a regular expression for the language accepted by the machine shown in Figure 1.

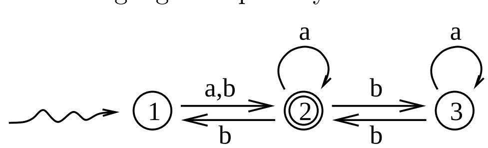


Figure 1

8. Give a grammar for the language accepted by the NFA shown in Figure 2 below.

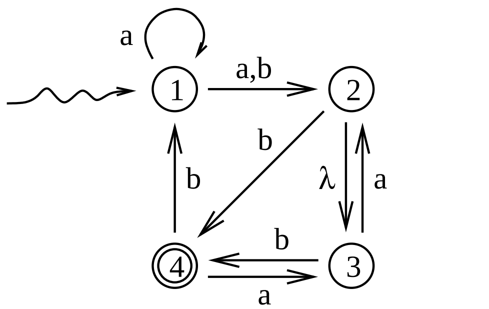


Figure 2: NFA for Problem 8

$S \to aS$
$S \to aA$
$S \to bA$
$A \to B$
$A \to bC$
$B \to bC$
$B \to aA$
$C \to aB$
$C \to bS$
$C \to \lambda$

10. For each of the binary languages described here, identify which of grammars (each with start symbol S) listed below generates that language?

(i) (d) All binary strings.

(ii) (e) All binary numerals for multiples of three.

(iii) (b) All binary strings which have the same number of 0's as 1's.

(iv) (a) All binary strings w which have the Dyck property, that is, w has equal number of 0's and 1's, and each prefix of w has at least as many 1's as 0's.

(v) (c) All binary numerals for powers of two.

(vi) (f) The language accepted by the NFA shown in Figure 3 below.

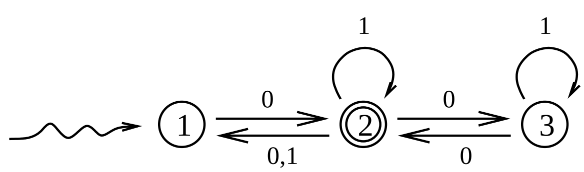


Figure 3

(a) $S \to 1S0S \mid \lambda$

(b) $S \to AS \mid BS \mid \lambda$
$A \to 1A0A \mid \lambda$
$A \to 0B1B \mid \lambda$

(c) $S \to 1A$
$A \to 0A \mid \lambda$

(d) $S \to 1S \mid 0S \mid \lambda$

(e) $S \to 1A \mid 0$
$A \to 1B \mid 0C$
$B \to 0B \mid 1A \mid \lambda$
$C \to 1C \mid 0A$

(f) $S \to 0A$
$A \to 0S \mid 1S \mid 0B \mid 1A \mid \lambda$
$B \to 1B \mid 0A$

11. Each of these regular expressions is for one of the languages given in Problem 10 above. Identify the correct language for each regular expression.

A. (iv) or (a) 10^*

B. (vi) or (f) $0(1 + (0 + 1)0 + 01^*0)^*$

C. (ii) or (e) $0 + 1(01^*0)^*1(1(01^*0)^*1)^*$ (I had the wrong expression!)

D. (i) or (d) $(0 + 1)^*$

12. What are the four language (or grammar) classes of the Chomsky hierarchy? Be sure to mention the type numbers as well as the name of the class.

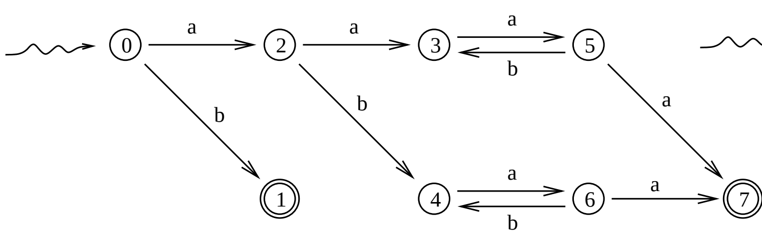
Type 0: unrestricted grammars, recursively enumerable languages.

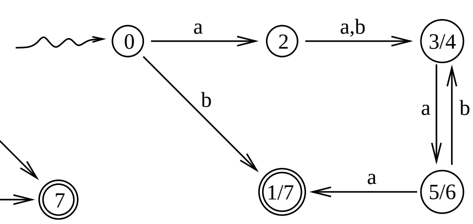
Type 1: context-sensitive grammars, context-sensitive languages.

Type 2: context-free gramars, context-free languages.

Type 3: regular grammars, regular languages.

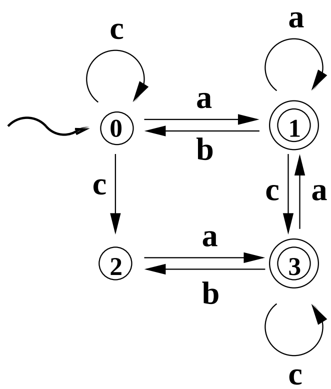
13. Draw a minimal DFA equivalent to the DFA shown above

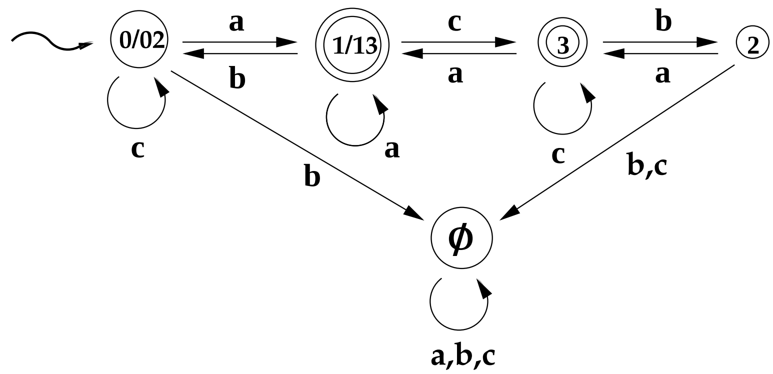




	0	1	2	3	4	5	6	7
0	0	X	X	X	X	X	X	X
1	X	0	X	X	X	X	X	0
2	X	X	0	X	X	X	X	X
3	X	X	X	0	0	X	X	X
4	X	X	X	0	0	X	X	X
5	X	X	X	X	X	0	0	X
6	X	X	X	X	X	0	0	X
7	X	0	X	X	X	X	X	0

14. Draw a minimal DFA equivalent to the NFA shown in Figure 4. Show your work.





	a	b	c
0	1	$\emptyset$	02
1	1	0	3
2	3	$\emptyset$	$\emptyset$
3	1	2	3
02	13	$\emptyset$	3
13	1	2	3
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$