CS 456/656 Study Guide for Examination March 6, 2024

A *binary language* is a subset of Σ^*, where $\Sigma = \{0, 1\}$, and a binary function is a function $\Sigma^* \to \Sigma^*$. The *unary* alphabet is $\{1\}$. The first written numerals were probably written in unary. We still use those numerals today.

1. True or False. T = true, F = false, and O = open, meaning that the answer is not known science at this time.

   (i) ________ Every subset of a regular language is regular.
   (ii) ________ The complement of a CFL is always a CFL.
   (iii) ________ The class of context-free languages is closed under union.
   (iv) ________ The class of context-free languages is closed under intersection.
   (v) ________ The set of binary numerals for multiples of 23 is regular.
   (vi) ________ The set of binary numerals for prime numbers is in $\mathcal{P}$–TIME.
   (vii) ________ Every PDA is equivalent to some DPDA.
   (viii) ________ Every language is countable.
   (ix) The set of languages over the binary alphabet is countable.
   (x) $\mathcal{P} = \mathcal{NP}$.
   (xi) The complement of any $\mathcal{P}$–TIME language is $\mathcal{P}$–TIME.
   (xii) The complement of any $\mathcal{NP}$ language is $\mathcal{NP}$.
   (xiii) The complement of any decidable language is decidable.
   (xiv) The complement of any undecidable language is undecidable.

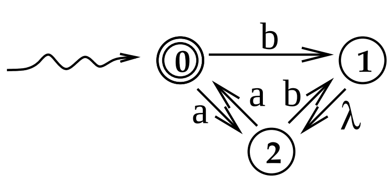
2. Give an unambiguous CFG which generates a language not accepted by any DPDA.

3. Suppose L is a problem such that you can check any suggested solution in polynomial time. Which one of these statements is certainly true?

   (i) L is $\mathcal{P}$.
   (ii) L is $\mathcal{NP}$.
   (iii) L is $\mathcal{NP}$-complete.

4. L be the language of all binary strings in which each 0 is followed by 1. Draw a DFA which accepts L.

5. Consider the NFA M pictured below.



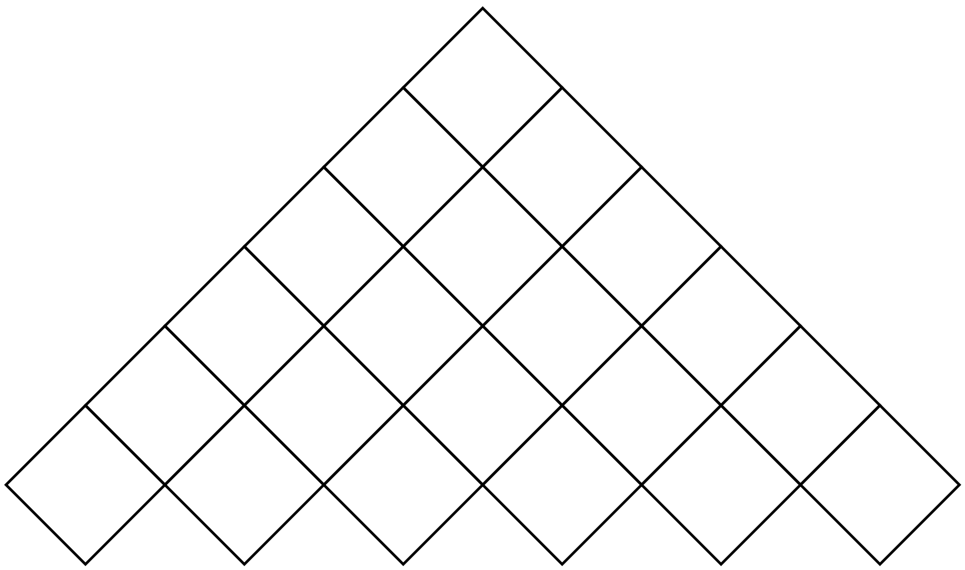
Construct a minimal DFA equivalent to M.

6. Let G_1 be the CF grammar given below. Prove that G_1 is ambiguous by giving two different parse trees for the string $iiwaea$.

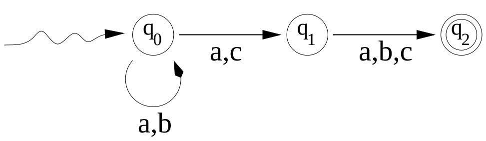
   1. $S \to a$
   2. $S \to wS$
   3. $S \to iS$
   4. $S \to iSeS$

7. The CNF grammar G_2, given below, is equivalent to the grammar G_1 given in Problem 6. Use the CYK algorithm to prove that $iiwaea$ is generated by G_2.

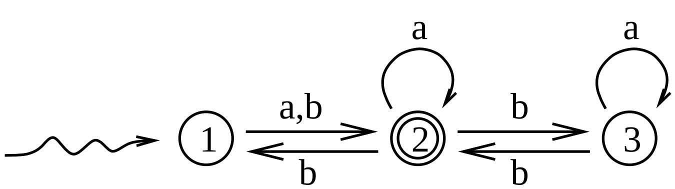
   1. $S \to a$
   2. $S \to WS$
   3. $W \to w$
   4. $S \to IS$
   5. $S \to AB$
   6. $A \to IS$
   7. $B \to ES$
   8. $E \to e$
   9. $I \to i$



8. Give a grammar, with at most 3 variables, for the language accepted by the follwing NFA.



9. Give a regular expression for the language accepted by the following NFA



10. Let L be the language consisting of all strings over $\{a, b\}$ which have equal numbers of each symbol. Give a CFG for L.

11. Design a DPDA which accepts the language described in Problem 10.

12. True or False. If the question is currently open, write "O" or "Open."

(i) ________ Every subset of a regular language is regular.

(ii) ________ $\mathcal{P} = \mathcal{NP}$.

(iii) ________ The complement of any $\mathcal{P}$-TIME language is $\mathcal{P}$-TIME.

(iv) ________ The complement of any $\mathcal{NP}$ language is $\mathcal{NP}$.

(v) ________ The complement of any $\mathcal{P}$–SPACE language is $\mathcal{P}$–SPACE. (**Think.**

(vi) ________ The complement of every recursive language is recursive.

(vii) ________ The complement of every recursively enumerable language is recursively enumerable.

(viii) ________ If a language L is recognized by an NFA with n states, then L has pumping length n.

(ix) ________ Given any unambiguous context-free grammar G and any string $w \in L(G)$, there is always a unique leftmost derivation of w using G.

(x) ________ For any deterministic finite automaton, there is always a unique minimal non-deterministic finite automaton equivalent to it.

(xi) ________ The union of any two context-free languages is context-free.

(xii) ________ The class of languages accepted by non-deterministic push-down automata is the same as the class of languages accepted by deterministic push-down automata.

(xiii) ________ Let π be the ratio of the circumference of a circle to its diameter. Then π is recursive.

(xiv) ________ The Kleene closure of any recursive language is recursive.

(xv) ________ If $\mathcal{P} = \mathcal{NP}$, then all one-way encoding systems are breakable in polynomial time.

(xvi) ________ A language L is in $\mathcal{NP}$ if and only if there is a polynomial time reduction of L to SAT.

(xvii) ________ The intersection of any context-free language with any regular language is context-free.

(xviii) ________ Let L be the set of all strings of the form $\langle G_1 \rangle \langle G_2 \rangle$ where G_1 and G_2 are equivalent context-free grammars. Then L is recursively enumerable.

(xix) ________ If L_1 reduces to L_2 in polynomial time, and if L_2 is $\mathcal{NP}$, and if L_1 is $\mathcal{NP}$-complete, then L_2 must be $\mathcal{NP}$-complete.

(xx) ________ The question of whether two regular expressions are equivalent is $\mathcal{NP}$-complete. (Do not guess. Look it up.)

(xxi) ________ The intersection of any two context-free languages is context-free.

(xxii) ________ If L_1 reduces to L_2 in polynomial time, and if L_2 is $\mathcal{NP}$, then L_1 must be $\mathcal{NP}$.

(xxiii) ________ Every language which is accepted by some non-deterministic machine is accepted by some deterministic machine.

(xxiv) ________ The language of all regular expressions over the alphabet $\{a, b\}$ is a regular language.

(xxv) ________ The equivalence problem for C++ programs is recursive.

(xxvi) ________ Every function that can be mathematically defined is recursive.

(xxvii) ________ A language is L is $\mathcal{NP}$ if and only if there is a polynomial time reduction of L to Boolean satisfiability.

(xxviii) ________ If there is a recursive reduction of the halting problem to a language L, then L must be undecidable.

(xxix) ________ If there is a recursive reduction of a language L to the halting problem, then L must be undecidable.

(xxx) ________ The set of rational numbers is countable.

(xxxi) ________ The set of real numbers is countable.

(xxxii) ________ The set of recursive real numbers is countable.

(xxxiii) ________ There are countably many binary functions.

(xxxiv) ________ There are countably many recursive binary functions.

13. Give a context-free language whose complement is not context-free.

14. State the pumping lemma for regular languages.

15. State the Church-Turing thesis. Why is it important?

16. Explain the verification definition of $\mathcal{NP}$.

17. Give a polynomial time reduction of the subset sum problem to the partition problem.

18. Prove that every decidable language can be enumerated by some machine in canonical order.

19. Prove that every language which can be enumerated in canonical order by some machine is recursive.

20. Prove that every language which can be enumerated by any machine is recognized by some machine.

21. True or False. T = true, F = false, and O = open, meaning that the answer is not known science at this time.

(i) ______ The context-free grammar equivalence problem is co-$\mathcal{RE}$.

(ii) ______ Let $L = \{(\langle G_1 \rangle, \langle G_2 \rangle)\}$: G_1 and G_2 are not equivalent. Then L is recursively enumerable.

(iii) ______ The factoring problem for unary numerals is $\mathcal{P}$–TIME

(iv) ______ The set of all binary numerals for prime numbers is in $\mathcal{P}$–TIME.

(v) ______ If L is a recursively enumerable language, there must be a machine which enumerates L in canonical order.

(vi) ______ The set of all positive real numbers is countable.

(vii) ______ For any alphabet Σ, the set of all recursively enumerable languages over Σ is countable.

(viii) ______ If L is a context-free language over the unary alphabet, then L must be regular.

(ix) ______ The union of any two undecidable languages is undecidable.

(x) ______ co-$\mathcal{P}$–TIME= $\mathcal{P}$–TIME.

22. Give a definition of a recursive real number. (There is more than one correct definition.)

23. Which of these languages (problems) are **known** to be $\mathcal{NP}$-complete? If a language, or problem, is known to be $\mathcal{NP}$-complete, fill in the first circle. If it is either known not to be $\mathcal{NP}$-complete, or if whether it is $\mathcal{NP}$-complete is not known at this time, fill in the second circle.

○ ○ Boolean satisfiability.
○ ○ 2–SAT.
○ ○ 3–SAT.
○ ○ Subset sum problem.
○ ○ Traveling salesman problem.
○ ○ C++ program equivalence.
○ ○ Partition.
○ ○ Regular language membership problem.
○ ○ Block sorting.

24. State the pumping lemma for context-free languages.

25. Give a polynomial time reduction of 3SAT to the independent set problem. (Pictures help.)