

CSC 456/656 Spring 2024 First Examination February 7 2024

Name:-----

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided. If you want a scratch page to be graded, write “See scratch paper,” on the test, and write your name on the scratch paper.

The entire examination is 240 points.

1. True or False. T = true, F = false, and O = open, meaning that the answer is not known to science at this time. (5 points each)
 - (i) **F** Every subset of a regular language is regular.
 - (ii) **F** The class of regular languages is closed under intersection.
 - (iii) **O** $\mathcal{P}\text{-TIME} = \mathcal{NP}$.
 - (iv) **T** The class of regular languages is closed under Kleene closure.
 - (v) **T** The class of context-free languages is closed under union.
 - (vi) **F** The class of context-free languages is closed under intersection.
 - (vii) **F** The set of binary numerals for prime numbers is a regular language.
 - (viii) **F** The Kleene closure of the empty language is empty.
 - (ix) **T** The complement of any $\mathcal{P}\text{-TIME}$ language is $\mathcal{P}\text{-TIME}$.
 - (x) **F** The complement of any context-free language is context-free.
 - (xi) **T** The complement of any recursive (that is, decidable) language is recursive.
 - (xii) **T** If Σ is an alphabet, then Σ^* is a regular language.
 - (xiii) **F** If L is a language and L^* is a regular language, then L must be a regular language.
 - (xiv) **T** In regular expressions, concatenation distributes over union.
 - (xv) **T** The regular grammar equivalence problem is decidable.
 - (xvi) **F** The context-free grammar equivalence problem is decidable.
 - (xvii) **T** The regular expression equivalence problem is decidable.
 - (xviii) **F** The language of palindromes over $\{a, b\}$ is not accepted by any PDA.
 - (xix) **F** Any context-free language is generated by some unambiguous context-free grammar.
 - (xx) **T** The Dyck language is context-free.

(xxi) **T** The complement of $L = \{a^n b^n c^n : n \geq 0\}$ is context-free. (We never did this in class, but if you think, you can figure it out using material we have covered.)

(xxii) **F** Every language is accepted by some machine.

(xxiii) **T** Let $F(n)$ be the n^{th} decimal digit of π after the decimal point. Then F is recursive.

2. [5 points] A language is context-free if and only if it is accepted by some **PDA**.

3. [5 points] Give an example of a language which is context-free but not regular.

$$\{a^n b^n : n \geq 0\}$$

4. [5 points] Give an example of a language which is not context-free.

$$\{a^n b^n c^n : n \geq 0\}$$

5. [20 points] Let G be the CF grammar given below, where S is the start symbol. G is ambiguous by giving two different **leftmost** derivations (not parse trees) for some string generated by G .

1. $S \rightarrow iS$

2. $S \rightarrow iSeS$ $S \Rightarrow iS \Rightarrow iiSeS \Rightarrow iiaeS \Rightarrow iiaea$

3. $S \rightarrow wS$ $S \Rightarrow iSeS \Rightarrow iiSeS \Rightarrow iiaeS \Rightarrow iiaea$

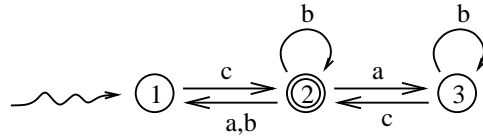
4. $S \rightarrow a$

6. [15 points] Given an unambiguous context-free grammar for the Dyck language, where (for ease of grading) you write “a” instead of “(” and “b” instead of “)” Partial credit for an ambiguous grammar.

$$S \rightarrow aSbS$$

$$S \rightarrow \lambda$$

7. [15 points] Write a regular expression for the language accepted by the machine shown below.



$$c(b + (a + b)c + ab^*c)^*$$

8. [20 points] What are the four language (or grammar) classes of the Chomsky hierarchy? Be sure to mention the type numbers as well as the name of the class.

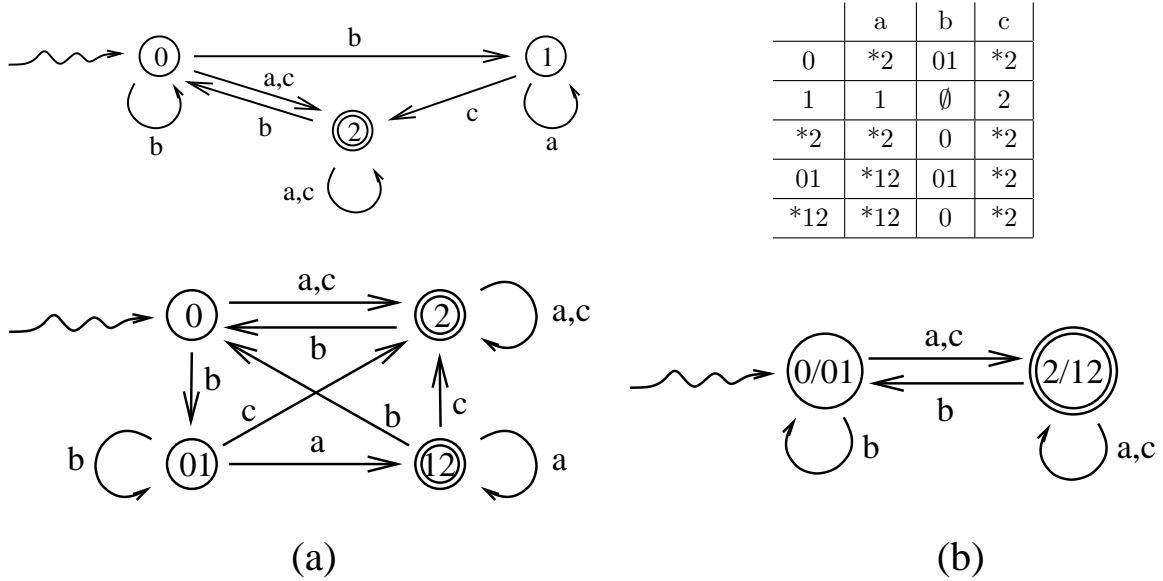
Type 0: Unrestricted grammars, recursively enumerable languages.

Type 1: Context-sensitive grammars, context-sensitive languages.

Type 2: Context-free grammars, context-free languages.

Type 3: Regular, or left-linear, or right-linear, grammars; regular languages.

9. [20 points] Find a minimal DFA equivalent to the NFA shown below.



In the table * indicates a final state. The states 1, 02 and 012 are unreachable, hence useless, in the DFA.

10. (a) [15 points] What language does the DFA below accept? (Hint: Arithmetic property.)

Binary numerals for positive integers equivalent to 1 modulo 3, *i.e.* $\{1, 100, 111, 1010, \dots\}$.

- (b) [15 points] Write a regular expression for that language.

$1(10^*1 + 01^*0)^*$

