Computer Science 456/656 Spring 2007 Second Examination, April 12, 2007

Name:____

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

The entire examination is 200 points.

1. True or False. [5 points each] In each blank space, write "T" if the statement is known to be true, "F" if the statement is known to be false, and "O" (for *open*) if it is not known to science at this time whether the statement is true.

   (a) _____ Every subset of a regular language is regular.

   (b) _____ There exists a machine[1] that runs forever and outputs the string of decimal digits of π (the well-known ratio of the circumference of a circle to its diameter).

   (c) _____ For every real number x, there exists a machine that runs forever and outputs the string of decimal digits of x.

   (d) _____ If a language has an unambiguous context-free grammar, then there must be some DPDA that accepts it.

   (e) _____ The problem of whether two given context-free grammars generate the same language is decidable.

   (f) _____ The problem of whether a given string is generated by a given context-free grammar is decidable.

   (g) _____ The language $\{a^n b^n c^n d^n \mid n \geq 0\}$ is recursive.

   (h) _____ Let L be the language over $\{a, b, c\}$ consisting of all strings which have more a's than b's and more b's than c's. There is some PDA that accepts L.

   (i) _____ There exists a mathematical proposition that can be neither proved nor disproved.

[1] As always in automata theory, "machine" means abstract machine, a mathematical object whose memory and running time are **not** constrained by the size and lifetime of the known (or unknown) universe, or any other physical laws. If we want to discuss the kind of machine that exists (or could exist) physically, we call it a "physical machine."

2. Fill the blanks. [5 points each blank]

   (a) An LALR ______________________ outputs a ______________________ derivation.

   (b) An ______________________ of a language L is a machine that outputs all the strings of L and no other strings.

3. [20 points] State the pumping lemma for context-free languages.

4. [30 points] State the Church-Turing thesis, and explain (in about 5 lines or less) why it is important.

5. [30 points]

1. $S \rightarrow \epsilon$
2. $S \rightarrow a_2 S_3 b_4 S_5$

	a	b	**eof**	S
0				1
1			**halt**	
2				
3		$s4$		
4		$r1$		
5				

Complete the ACTION and GOTO tables of an LALR parser for the grammar given above. This grammar unambiguously generates the "balanced parentheses" language, where a represents a left parenthesis, and b represents a right parenthesis. Example strings include ϵ, ab, $aabb$, $abab$, and $aabbab$.

6. [30 points] Give an implementation-level description of a Turing machine that decides the language $L = \left\{ w \in \{0,1\}^* \mid w \text{ contains twice as many 0s as 1s} \right\}$.

7. [30 points] What is the diagonal language? Give a brief sketch of the proof that it is not accepted by any Turing machine. (If you use more than this page, you are writing too much.)