Computer Science 456/656 Fall 2007 First Examination, September 27, 2007

Name:____

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

The entire examination is 215 points.

1. True or False. [5 points each]

   (a) _______ Every subset of a regular language is regular.

   (b) _______ Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^n b^n$, where $n \geq 0$. Then L is a regular language.

   (c) _______ The complement of any regular language is regular.

   (d) _______ Every context-free language that has an unambiguous context-free grammar is accepted by some deterministic push-down automaton.

   (e) _______ The language consisting of all base three numerals for prime numbers is regular.

   (f) _______ There is a finite state machine that runs all valid Java programs.

   (g) _______ The union of any two regular languages is regular.

   (h) _______ The language consisting of all hexadecimal numerals for positive integers n such that $n \% 5 = 1$ is regular.

   (i) _______ The language consisting of all hexadecimal numerals for positive integers n such that $n^2 \% 5 = 1$ is regular.

   (j) _______ The complement of any context-free language is context-free.

2. [5 points each blank] Fill in the blanks.

   (a) Name two classes of machines that accept the class of regular languages. ________________________

   and ________________________.

   (b) Name one class of machines that accepts the class of context-free languages. ________________________.

   (c) A ________________________ parser gives a *reverse rightmost* derivation of its input string.

3. [25 points] Draw a DFA which accepts the language of all strings over $\Sigma = \{0, 1\}$ which have an odd number of ones, and which contain the substring 00.

4. [25 points] Write a regular expression for the language accepted by the following DFA:

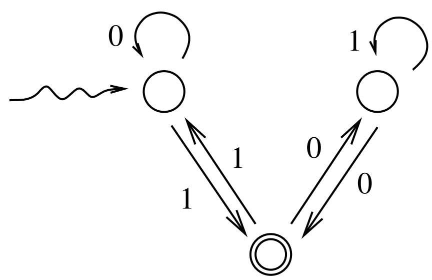

Figure 1: Find a Regular Expresion

5. [15 points] State the pumping lemma. The space below is enough. If you go over that space, either your writing is extremely large, or you're writing too much.

6. [20 points] Prove that the following grammar is ambiguous by giving two different leftmost derivations, or two different parse trees, for the string $iiaea$. The start symbol is S.

   (a) $S \rightarrow iS$

   (b) $S \rightarrow iSeS$

   (c) $S \rightarrow a$

7. [30 points] Consider the NFA whose transition diagram is drawn below, where the input alphabet is $\{a, b, c\}$. Draw the transition diagram of an equivalent minimal DFA. Show your steps.

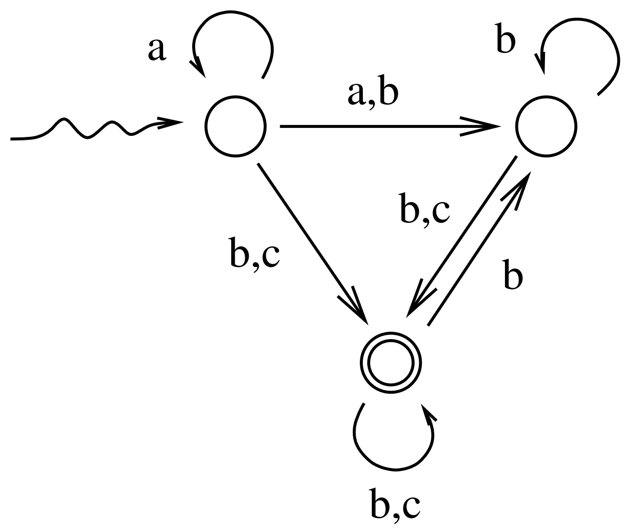

Figure 2: Find a minimal DFA equivalent to this NFA

8. [30 points] Let $L = \left\{ w \in \{a,b\}^* \mid \#_a(w) = \#_b(w) \right\}$, where $\#_a(w)$ denotes the number of instances of the symbol a in the string w. For example, $aababbba \in L$. Design a PDA that accepts L. It is not required for you to give a context-free grammar for L.

There are many rather different correct answers for this problem.