Computer Science 456/656 Fall 2008 First Examination, September 25, 2008

Name:_____

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

The entire examination is 215 points.

1. True or False. [5 points each]

   (a) _____ Every subset of a regular language is regular.

   (b) _____ Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^n b^n$, where $n \geq 0$. Then L is a regular language.

   (c) _____ The complement of any regular language is regular.

   (d) _____ Every context-free language that has an unambiguous context-free grammar is accepted by some deterministic push-down automaton.

   (e) _____ The language consisting of all base three numerals for prime numbers is regular.

   (f) _____ There is a finite state machine that runs all valid Java programs.

   (g) _____ The union of any two regular languages is regular.

   (h) _____ The language consisting of all hexadecimal numerals for positive integers n such that $n^2 \ \% \ 5 = 1$ is regular.

   (i) _____ The complement of any context-free language is context-free.

   (j) _____ Every context-free language is in the class $\mathcal{P}$, the class of polynomial time languages.

2. [5 points each blank] Fill in the blanks.

   (a) Name two classes of machines that accept the class of regular languages. _____

   and _____.

   (b) Name one class of machines that accepts the class of context-free languages. _____.

   (c) If a machine M is _____, there is at most one legal move M can make from any give configuration.

3. [25 points] Draw an NFA with four states which accepts the language described by the regular expression $(a+b)^*a(a+b)(a+b)$.

4. [25 points] Write a regular expression for the language accepted by the following DFA:

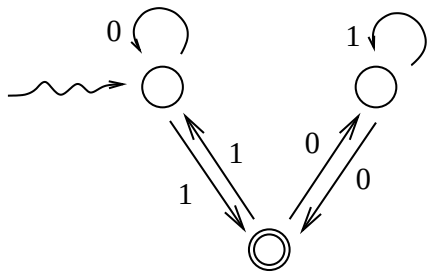

Figure 1: Find a Regular Expression

5. [15 points] State the pumping lemma. The space below is enough. If you go over that space, either your writing is extremely large, or you're writing too much.

6. [20 points] Prove that the following grammar is ambiguous by giving two different leftmost derivations, or two different parse trees, for the string $iiaea$. The start symbol is S.

   (a) $S \rightarrow iS$

   (b) $S \rightarrow iSeS$

   (c) $S \rightarrow a$

7. [30 points] Consider the NFA whose transition diagram is drawn below, where the input alphabet is $\{a, b, c\}$. Draw the transition diagram of an equivalent minimal DFA. Show your steps.

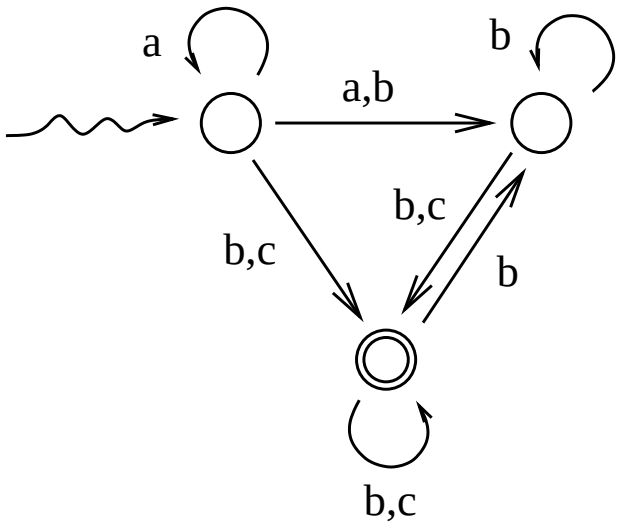

Figure 2: Find a minimal DFA equivalent to this NFA

8. [30 points] Let $L = \{w \in \{a,b\}^* \mid \#_a(w) = \#_b(w)\}$, here $\#_a(w)$ denotes the number of instances of the symbol a in the string w. For example, $aababbba \in L$, because that string has 3 a's and 3 b's. Give a context-free grammar for L.

   There are many rather different correct answers for this problem.