Computer Science 456/656 Fall 2008 Second Examination, November 6, 2008

Name:____

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

The entire examination is 225 points.

1. True or False. [5 points each] "T" if the statement is known to be true, "F" if the statement is known to be false, and "O" (for *open*) if it is not known to science at this time whether the statement is true.

   (a) _____ Every subset of a regular language is regular.

   (b) _____ There exists a machine that runs forever and outputs the string of decimal digits of π (the well-known ratio of the circumference of a circle to its diameter).

   (c) _____ For every real number x, there exists a machine that runs forever and outputs the string of decimal digits of x.

   (d) _____ If a language has an unambiguous context-free grammar, then there must be some DPDA that accepts it.

   (e) _____ The problem of whether a given context-free grammar generates a given string is in the class $\mathcal{P}$.

   (f) _____ A language L is decidable if and only if there is some machine that enumerates L.

   (g) _____ The problem of whether two given context-free grammars generate the same language is decidable.

   (h) _____ The language $\{a^n b^n c^n \mid n \geq 0\}$ is in the class $\mathcal{P}$.

   (i) _____ Let L be the language over $\{a, b, c\}$ consisting of all strings which have more a's than b's and more b's than c's. There is some PDA that accepts L.

   (j) _____ There exists a mathematical proposition that can be neither proved nor disproved.

   (k) _____ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a binary numeral.

   (l) _____ Every undecidable problem is $\mathcal{NP}$-complete.

   (m) _____ Every bounded function is recursive.

   (n) _____ For any two languages L_1 and L_2, if L_1 is $\mathcal{NP}$-complete, L_2 is $\mathcal{NP}$, and there is a polynomial time reduction of L_1 to L_2, then L_2 must be $\mathcal{NP}$-complete.

   (o) _____ If P is a mathematical proposition that can be stated using n binary bits, and P has a proof, then P must have a proof whose length is $O(2^{2^n})$.

2. [10 points] Give a definition of *context-sensitive grammar*.

3. [20 points] State the pumping lemma for context-free languages. (Your answer must be correct in its structure, not just the words you use. Even if all the correct words are there, you could get no credit if you get the logic wrong.)

4. [30 points] State the Church-Turing thesis, and explain (in about 5 lines or less) why it is important.

5. [30 points]

1. $S \rightarrow \epsilon$
2. $S \rightarrow a_2 S_3 b_4 S_5$

	a	b	**eof**	S
0				1
1			**halt**	
2				
3		$s4$		
4		$r1$		
5				

Complete the ACTION and GOTO tables of an LALR parser for the grammar given above. This grammar unambiguously generates the "balanced parentheses" language, where a represents a left parenthesis, and b represents a right parenthesis. Example strings include ϵ, ab, $aabb$, $abab$, and $aabbab$.

6. [30 points] Give a sketch of the proof that the independent set problem is $\mathcal{NP}$-complete, assuming that 3-CNF-SAT is $\mathcal{NP}$-complete. You may draw pictures and use examples.

7. [30 points] Give a general grammar that generates $L = \{1^{2^n}\}$, the language of all strings over the unary alphabet $\{1\}$ which have length a power of 2.