Computer Science 456/656 Fall 2010 First Examination, September 23, 2010

Name:_____

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

The entire examination is 235 points.

1. True or False. [5 points each]

   (a) _____ Every subset of a regular language is regular.

   (b) _____ Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^m b^n$, where $m, n \geq 0$. Then L is a regular language.

   (c) _____ The complement of every regular language is regular.

   (d) _____ The Kleene closure of every context-free language is context-free.

   (e) _____ If a language has an unambiguous context-free grammar, then it is is accepted by some deterministic push-down automaton.

   (f) _____ If a language has an ambiguous context-free grammar, then it is is not accepted by any deterministic push-down automaton.

   (g) _____ There is a PDA that accepts all valid C++ programs.

   (h) _____ The intersection of any two regular languages is regular.

   (i) _____ The language consisting of all base 7 numerals for positive integers n such that $n \ \% \ 3 = 2$ is regular.

   (j) _____ The intersection of any two context-free languages is context-free.

2. [5 points each blank] Fill in the blanks.

   (a) Name two classes of machines that accept the class of regular languages. _____

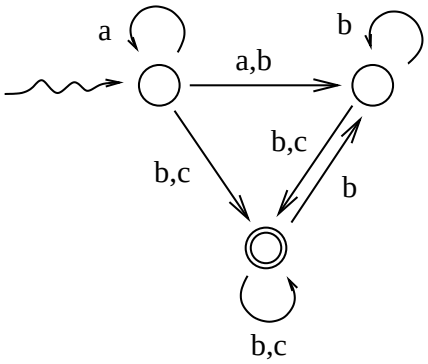
   and _____.

   (b) Name one class of machines that accepts the class of context-free languages. _____.

   (c) If a machine M is _____, there is at most one legal move M can make from any give configuration.

3. [25 points] Draw an NFA with five states which accepts the language described by the regular expression $(0+1)^*0(0+1)(0+1)(0+1)$

4. [25 points] Write a regular expression for the language accepted by the following NFA. If your answer is unnecessarily long by a wide margin, I might mark it wrong even if it's right.



Find a Regular Expression

5. [15 points] State the pumping lemma. The space below is enough. If you go over that space, either your writing is extremely large, or you're writing too much.

6. Let L be the language generated by the context-free grammar given in the first box. The second box contains a Chomsky Normal Form (CNF) grammar that also generates L.

(a) $S \to a$

(b) $S \to wS$

(c) $S \to iS$

(d) $S \to iSeS$

(a) $S \to WS$

(b) $S \to IS$

(c) $S \to AB$

(d) $A \to IS$

(e) $B \to ES$

(f) $S \to a$

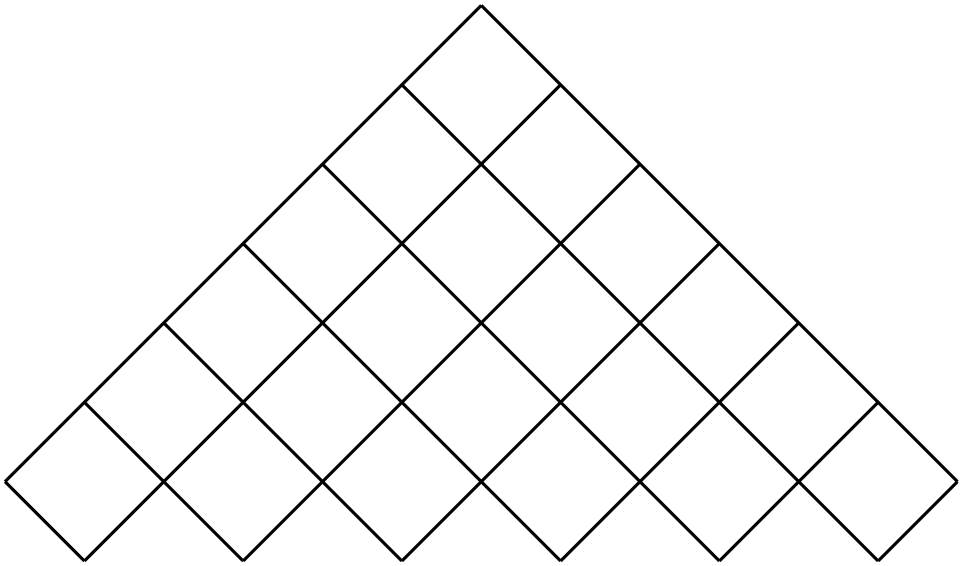
(g) $I \to i$

(h) $W \to w$

(i) $E \to e$

(a) [20 points] Use the CYK algorithm to prove that the string *iwiaea* is a member of L. Use the figure on the next page for your work.

(b) [20 points] By looking at your work carefully, you can determine that the CNF grammar given above is ambiguous. Write two different parse trees for *iwiaea*, using the CNF grammar.



i w i a e a

7. [30 points] Consider the NFA whose transition diagram is drawn below, where the input alphabet is $\{a, b, c\}$. Draw the transition diagram of an equivalent minimal DFA. Show your steps.

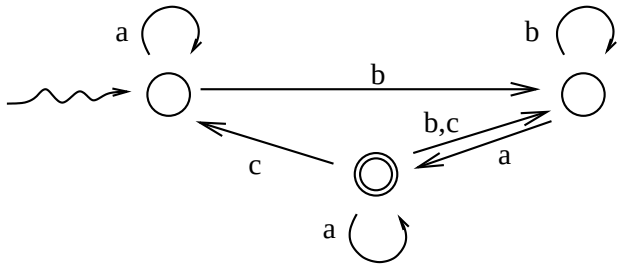


Figure 1: Find a minimal DFA equivalent to this NFA

8. [30 points] Let $L = \left\{w \in \{a,b\}^* \mid \#_a(w) = 2\#_b(w)\right\}$, here $\#_a(w)$ denotes the number of instances of the symbol a in the string w. For example, $aaababaaabba \in L$, because that string has the twice as many a's as b's. Give a context-free grammar for L. Your grammar may be ambiguous.