Name:\_\_\_\_\_

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

The entire examination is 215 points.

- 1. True or False. [5 points each] T = true, F = false, O = open, meaning that the answer is not known to science at this time.
  - (a) \_\_\_\_\_ Every subset of a regular language is regular.
  - (b)  $\_\__ \mathcal{P}\text{-TIME} = \mathcal{NP}\text{-TIME}.$
  - (c) \_\_\_\_\_ If a languages is not in the class  $\mathcal{P}$ -TIME, then it must be undecidable.
  - (d) \_\_\_\_\_ The set of all mathematical statements which are provably true is decidable.
  - (e) \_\_\_\_\_ The set of all mathematical statements which are provably true is recursively enumerable.
  - (f) \_\_\_\_\_ If a language L is accepted by some NTM, then there is a TM which accepts L.
  - (g) \_\_\_\_\_ The problem of whether a given grammar generates the empty language is decidable.
  - (h) \_\_\_\_\_ The problem of whether a given grammar generates all strings over an alphabet  $\Sigma$  is decidable.
  - (i) \_\_\_\_\_ Every language that can be parsed by an LALR parser can be accepted by some DPDA.
  - (j) \_\_\_\_\_ Every language generated by an unambiguous context-free grammar is accepted by some DPDA.
  - (k) \_\_\_\_\_ If a real number x is the solution to a polynomial equation with integral coefficients, then there must be a TM that runs forever, writing the decimal digits of x.
  - (l) \_\_\_\_\_ There exists a polynomial time algorithm which finds the factors of any positive integer, if the input integer is written in unary ("caveman") notation.
- 2. [5 points each blank] Fill in the blanks.

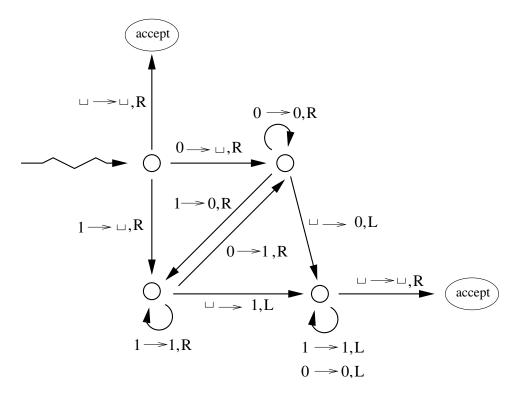
  - (b) If every string generated by a grammar G has a unique leftmost derivation using G, then we say that G is \_\_\_\_\_\_.
  - (c) We are able to prove that every language accepted by an NTM is accepted by a TM. In this proof, we construct a TM that emulates the NTM using a bitstring which we call a \_\_\_\_\_\_ string.

- 3. [30 points]
  - (a) What is the Church Turing Thesis?
  - (b) Why is it important?
- 4. [20 points] Let L be the language generated by the context-free grammar given below. Give a Chomsky normal form grammar which generates L.
  - $$\begin{split} 1. \ S &\to a \\ 2. \ S &\to wS \\ 3. \ S &\to iS \\ 4. \ S &\to iSeS \end{split}$$

5. [25 points] Write proof (as well as you can) of the statement that a language L can be enumerated by some machine in canonical order if L is recursive.

## 6. [25 points]

What does the Turing machine illustrated below do? If it takes more than one line of normal size printing to write your answer, it is almost certainly wrong.



7. [20 points] State the pumping lemma for context-free languages. The space below is enough. If you go over that space, either your writing is extremely large, or you're writing too much.

8. [20 points] Using pictures and words, give an informal explanation of how you can design a multi-track one-tape TM that emulates a multi-tape TM.