Computer Science 456/656 Spring 2009 Practice Examination for Second Examination, March 26, 2009

The entire examination is 290 points.

1. True or False. [5 points each] T = true, F = false, and O = open, meaning that the answer is not known to science at this time.

   (a) ______ There exists a machine[1] that runs forever and outputs the string of decimal digits of π (the well-known ratio of the circumference of a circle to its diameter).

   (b) ______ For every real number x, there exists a machine that runs forever and outputs the string of decimal digits of x.

   (c) ______ If a language has an unambiguous context-free grammar, then there must be some DPDA that accepts it.

   (d) ______ The problem of whether two given context-free grammars generate the same language is decidable.

   (e) ______ The problem of whether a given string is generated by a given context-free grammar is decidable.

   (f) ______ The language $\{a^n b^n c^n d^n \mid n \geq 0\}$ is recursive.

   (g) ______ Let L be the language over $\{a, b, c\}$ consisting of all strings which have more a's than b's and more b's than c's. There is some PDA that accepts L.

   (h) ______ There exists a mathematical proposition that can be neither proved nor disproved.

   (i) ______ The problem of whether a given context-sensitive grammar generates a given string is in the class $\mathcal{NP}$.

   (j) ______ The language $\{a^n b^n c^n \mid n \geq 0\}$ is in the class $\mathcal{P}$.

   (k) ______ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a binary numeral.

   (l) ______ Every undecidable problem is $\mathcal{NP}$-complete.

   (m) ______ The problem of whether a given context-free grammar generates all strings is decidable.

[1] As always in automata theory, "machine" means abstract machine, a mathematical object whose memory and running time are **not** constrained by the size and lifetime of the known (or unknown) universe, or any other physical laws. If we want to discuss the kind of machine that exists (or could exist) physically, we call it a "physical machine."

(n) ______ The language $\{a^n b^n \mid n \geq 0\}$ is context-free.

(o) ______ The language $\{a^n b^n c^n \mid n \geq 0\}$ is context-free.

(p) ______ The language $\{a^i b^j c^k \mid j = i + k\}$ is context-free.

(q) ______ The intersection of any three regular languages is context-free.

(r) ______ If a language L is undecidable, then there can be no machine that enumerates L.

(s) ______ **(Warning: this one is hard.)** If f is any function on positive integers, there must be a recursive function g such that $f(n) = O(g(n))$.

(t) Recall that if $\mathcal{L}$ is a class of languages, co-$\mathcal{L}$ is defined to be the class of all languages that are not in $\mathcal{L}$.

______ Let $\mathcal{RE}$ be the class of all recursively enumerable languages. If L is in $\mathcal{RE}$ and also L is in co-$\mathcal{RE}$, then L must be decidable.

(u) ______ Every problem that can be mathematically defined has an algorithmic solution.

(v) ______ If a language has an unambiguous context-free grammar, then there must be some DPDA that accepts it.

(w) ______ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a unary numeral.

(x) ______ Every bounded function is recursive.

(y) ______ For any two languages L_1 and L_2, if L_1 is $\mathcal{NP}$-complete, L_2 is $\mathcal{NP}$, and there is a polynomial time reduction of L_1 to L_2, then L_2 must be $\mathcal{NP}$-complete.

(z) ______ If P is a mathematical proposition that can be stated using n binary bits, and P has a proof, then P must have a proof whose length is $O(2^{2^n})$.

Fill the blanks. [5 points each blank]

(a) An LALR ____________________ outputs a ____________________ derivation.

(b) An ____________________ of a language L is a machine that outputs all the strings of L and no other strings.

(c) An LALR parser outputs a ____________________ derivation.

(d) An LR parser is a ____________________ parser.

(e) If a language is accepted by some Turing machine, it is ____________________ enumerable.

(f) An LL (top-down) parser outputs a ____________________ derivation.

2. [10 points] Give a definition of *context-sensitive grammar*.

3. [20 points] State the pumping lemma for context-free languages. (Your answer must be correct in its structure, not just the words you use. Even if all the correct words are there, you could get no credit if you get the logic wrong.)

4. [30 points] State the Church-Turing thesis, and explain (in about 5 lines or less) why it is important.

5. [30 points]

   1. $S \to \epsilon$

   2. $S \to a_2 S_3 b_4 S_5$

	a	b	**eof**	S
0				1
1			**halt**	
2				
3		$s4$		
4		$r1$		
5				

Complete the ACTION and GOTO tables of an LALR parser for the grammar given above. This grammar unambiguously generates the "balanced parentheses" language, where a represents a left parenthesis, and b represents a right parenthesis. Example strings include ϵ, ab, $aabb$, $abab$, and $aabbab$.

6. [30 points] Give a sketch of the proof that the independent set problem is $\mathcal{NP}$-complete, assuming that 3-CNF-SAT is $\mathcal{NP}$-complete. You may draw pictures and use examples.

7. [30 points] Give a general grammar that generates $L = \{1^{2^n}\}$, the language of all strings over the unary alphabet $\{1\}$ which have length a power of 2.

8. [30 points] Give an implementation-level description of a Turing machine that decides the language $L = \{w \in \{0,1\}^* \mid w \text{ contains twice as many 0s as 1s}\}$.

9. [30 points] What is the diagonal language? Give a brief sketch of the proof that it is not accepted by any Turing machine. (If you use more than this page, you are writing too much.)

10. [30 points] Find a Chomsky Normal Form grammar equivalent to the following context-free grammar.

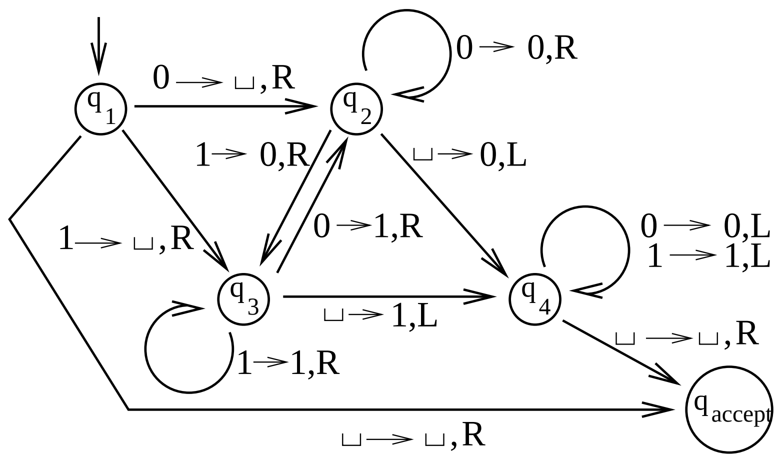
$S \to S \mid SAS \mid B$

$A \to b \mid B \mid aB$

$B \to c$

$B \to \varepsilon$

11. [20 points] Describe, in English, what the Turing machine diagrammed below does. Hint: It only takes a few words. If you cannot explain it in the space alloted below, you do not have the right answer.



12. [20 points] Let $\Sigma = \{1\}$. Consider the grammar whose terminals are Σ, whose variables are $\{S, A, B, C\}$, where S is the start symbol, and whose productions are:

1. $S \to A1B$
2. $A \to AC$
3. $C1 \to 11C$
4. $CB \to B$
5. $A \to \epsilon$
6. $B \to \epsilon$

What language over Σ does G generate?

13. [30 points] Prove that, if a language L is decidable, then L can be enumerated in canonical order by some machine.

14. [30 points] Give a brief explanation of why any language accepted by an NTM is accepted by some TM.