Computer Science 456/656 Spring 2009 First Examination, February 12, 2009

Name:_____

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

The entire examination is 215 points.

1. True or False. [5 points each]

   (a) _____ Every subset of a regular language is regular.

   (b) _____ Let L be the language over $\Sigma = \{a, b\}$ consisting of all strings of the form $a^n b^m$, where $n, m \geq 0$. Then L is a regular language.

   (c) _____ The complement of any regular language is regular.

   (d) _____ Every context-free language that has an ambiguous context-free grammar also has an unambiguous context-free grammar.

   (e) _____ The language consisting of all base three numerals for prime numbers is regular.

   (f) _____ The programming language Java is context-free.

   (g) _____ The Kleene closure of any context-free language is context-free.

   (h) _____ The language consisting of all binary numerals for positive integers which are perfect squares, *e.g.*, 0, 1, 100, 1001, ... is regular.

   (i) _____ The intersection of any two context-free languages is context-free.

   (j) _____ The reverse of any regular language is regular. (The *reverse* of a string w is defined to be the string w^R consisting of the same symbols in reverse order. For example, the reverse of "george" is "egroeg." If L is a language, then L^R, the *reverse* of L, is defined to be $\{w^R : w \in L\}$.)

2. [5 points each blank] Fill in the blanks.

   (a) Name two classes of machines that accept the class of regular languages. _____

   and _____.

   (b) Name one class of machines that accepts the class of context-free languages. _____.

   (c) Two grammars are _____ if they generate the same language.

3. [25 points] Draw an NFA with three or four states which accepts the language described by the regular expression $a((b + \varepsilon)a)^*b$. Full credit if you can do it with three states, partial credit for four states.

4. [25 points] Write a regular expression for the language accepted by the following DFA:

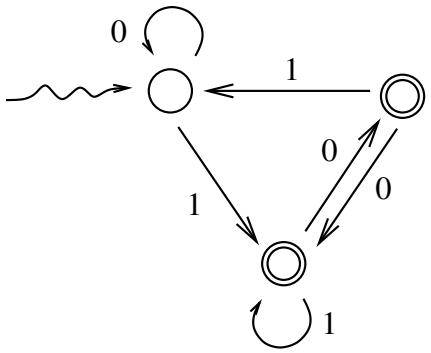

Figure 1: Find a Regular Expression

5. [15 points] State the pumping lemma. The space below is enough. If you go over that space, either your writing is extremely large, or you're writing too much.

6. [20 points] Prove that the following grammar is ambiguous by giving two different leftmost derivations for the string $ab + c$. The start symbol is E.

   (a) $E \to E + E$

   (b) $E \to EE$

   (c) $E \to a$

   (d) $E \to b$

   (e) $E \to c$

7. [30 points] Consider the NFA whose transition diagram is drawn below, where the input alphabet is $\{a, b, c\}$. Draw the transition diagram of an equivalent minimal DFA. Show your steps.

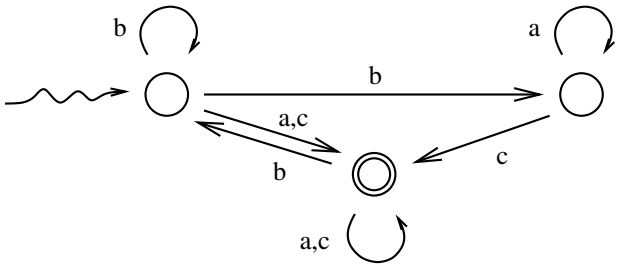

Figure 2: Find a minimal DFA equivalent to this NFA

8. [30 points] Let $L = \{w \in \{a,b\}^* \mid \#_a(w) \leq \#_b(w)\}$, here $\#_a(w)$ denotes the number of instances of the symbol a in the string w. Give a context-free grammar for L.

   There are many rather different correct answers for this problem. Choose one that is as simple as you can.