The entire examination is 235 points.

1. True or False. [5 points each] T = true, F = false, and O = open, meaning that the answer is not known to science at this time.

   (a) ______ Every subset of a regular language is regular.

   (b) ______ The complement of every \( \mathcal{NP} \) language is \( \mathcal{NP} \).

   (c) ______ The complement of every recursive language is recursive.

   (d) ______ The complement of every recursively enumerable language is recursively enumerable.

   (e) ______ The set of all binary numerals for prime numbers is in \( \mathcal{P} \).

   (f) ______ Every context-free language is accepted by some deterministic machine.

   (g) ______ The problem of whether two given context-free grammars generate the same language is decidable.

   (h) ______ Suppose a machine \( M \) accepts a language \( L_1 \), and \( L_2 \) is a proper subset of \( L_1 \). Then, \( M \) accepts \( L_2 \).

2. Fill the blanks. [5 points each blank]

   (a) A machine \( M \) is ______________________ if, given any configuration \( x \) of \( M \), there is at most one configuration \( y \) such that \( x \mapsto y \).

   (b) A context-free grammar \( G \) is ______________________ if there is some string in \( L(G) \) which has more than one parse tree.

   (c) If a language \( L_1 \) can be easily reduced to a language \( L_2 \), then \( L_1 \) is at least as _________ as \( L_2 \).

      (Hint: the answer is either “hard” or “easy.”)

3. [20 points] Give the definition of \( \mathcal{NP} \)-time. (Yes, it is possible to write it in this space.)
4. [30 points]

1. $S \rightarrow \epsilon$

2. $S \rightarrow a_{2}S_{3}b_{4}S_{5}$

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Complete the ACTION and GOTO tables of an LALR parser for the grammar given above. This grammar unambiguously generates the “balanced parentheses” language, where $a$ represents a left parenthesis, and $b$ represents a right parenthesis. Example strings include $\epsilon$, $ab$, $aabb$, $abab$, and $aabbab$. 
5. [40 points] We can prove that the independent set problem is $\mathcal{NP}$-complete by reducing 3-CNF-Sat to it. This reduction is a function $R$, where, if $E$ is a Boolean expression in 3-CNF form, then $R(E) = (G)(k)$, where $G$ is a graph and $k$ is a non-negative number, such that $E$ is satisfiable if and only if $G$ has an independent set of size $k$.

Let $E = (x + y + z) \cdot (\overline{x} + z + \overline{w}) \cdot (\overline{x} + y + \overline{w}) \cdot (x + \overline{z} + \overline{w}) \cdot (\overline{x} + \overline{z} + \overline{w})$, and let $R(E) = (G)(k)$.

(a) Draw a picture of the graph $G$.
(b) What is the value of $k$?
(c) Circle an independent set of size $k$ in your picture of $G$, and give a corresponding satisfying assignment of $E$. 
6. [30 points] Let $L$ be the language of all strings over the binary alphabet $\{0, 1\}$ which have an even number of 1's. Draw the diagram of a Turing Machine that accepts $L$. 
7. [30 points] The following general grammar generates the language of all strings over \( \{1\} \) of length a power of 2. Write a derivation of the string 1111 using this grammar.

1. \( S \rightarrow A1B \)
2. \( A \rightarrow AC \)
3. \( C1 \rightarrow 11C \)
4. \( CB \rightarrow B \)
5. \( A \rightarrow \epsilon \)
6. \( B \rightarrow \epsilon \)
8. [30 points] Prove that, if a language $L$ is decidable, then $L$ can be enumerated in canonical order by some machine.