**Computer Science 456/656 Spring 2009 Second Examination, March 26, 2009**

**The entire examination is 235 points.**

1. True or False. [5 points each] T = true, F = false, and O = open, meaning that the answer is not known to science at this time.
   (a) ______ Every subset of a regular language is regular.
   (b) ______ The complement of every $\mathcal{NP}$ language is $\mathcal{NP}$.
   (c) ______ The complement of every recursive language is recursive.
   (d) ______ The complement of every recursively enumerable language is recursively enumerable.
   (e) ______ The set of all binary numerals for prime numbers is in $\mathcal{P}$.
   (f) ______ Every context-free language is accepted by some deterministic machine.
   (g) ______ The problem of whether two given context-free grammars generate the same language is decidable.
   (h) ______ Suppose a machine $M$ accepts a language $L_1$, and $L_2$ is a proper subset of $L_1$. Then, $M$ accepts $L_2$.

2. Fill the blanks. [5 points each blank]
   (a) A machine $M$ is ______________________ if, given any configuration $x$ of $M$, there is at most one configuration $y$ such that $x \mapsto y$.
   (b) A context-free grammar $G$ is ______________________ if there is some string in $L(G)$ which has more than one parse tree.
   (c) If a language $L_1$ can be easily reduced to a language $L_2$, then $L_1$ is at least as ____________ as $L_2$. (Hint: the answer is either "hard" or "easy.")

3. [20 points] Give the definition of $\mathcal{NP}$-TIME. (Yes, it is possible to write it in this space.)

4. [30 points]

1. $S \to \epsilon$
2. $S \to a_2 S_3 b_4 S_5$

| | $a$ | $b$ | **eof** | $S$ |
|---|---|---|---|---|
| 0 | | | | 1 |
| 1 | | | **halt** | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |

Complete the ACTION and GOTO tables of an LALR parser for the grammar given above. This grammar unambiguously generates the "balanced parentheses" language, where $a$ represents a left parenthesis, and $b$ represents a right parenthesis. Example strings include $\epsilon$, $ab$, $aabb$, $abab$, and $aabbab$.

5. [40 points] We can prove that the independent set problem is $\mathcal{NP}$-complete by reducing 3-CNF-Sat to it. This reduction is a function $R$, where, if $E$ is a Boolean expression in 3-CNF form, then $R(E) = \langle G \rangle \langle k \rangle$, where $G$ is a graph and $k$ is a non-negative number, such that $E$ is satisfiable if and only if $G$ has an independent set of size $k$.

   Let $E = (x + y + z) * (!x + z + !w) * (!x + !y + w) * (x + !z + w) * (!x + !z + !w)$, and let $R(E) = \langle G \rangle \langle k \rangle$.

   (a) Draw a picture of the graph $G$.

   (b) What is the value of $k$?

   (c) Circle an independent set of size $k$ in your picture of $G$, and give a corresponding satisfying assignment of $E$.

6. [30 points] Let $L$ be the language of all strings over the binary alphabet $\{0, 1\}$ which have an even number of 1's. Draw the diagram of a Turing Machine that accepts $L$.

7. [30 points] The following general grammar generates the language of all strings over $\{1\}$ of length a power of 2. Write a derivation of the string 1111 using this grammar.

   1. $S \to A1B$
   2. $A \to AC$
   3. $C1 \to 11C$
   4. $CB \to B$
   5. $A \to \epsilon$
   6. $B \to \epsilon$

8. [30 points] Prove that, if a language $L$ is decidable, then $L$ can be enumerated in canonical order by some machine.