True/False Questions, Part I

1. True or False. T = true, F = false, and O = open, meaning that the answer is not known science at this time. In the questions below, $\mathcal{P}$ and $\mathcal{NP}$ denote $\mathcal{P}$-TIME and $\mathcal{NP}$-TIME, respectively.

   (i) _____ Let L be the language over $\{a, b, c\}$ consisting of all strings which have more a's than b's and more b's than c's. There is some PDA that accepts L.

   (ii) _____ The language $\{a^n b^n \mid n \geq 0\}$ is context-free.

   (iii) _____ The language $\{a^n b^n c^n \mid n \geq 0\}$ is context-free.

   (iv) _____ The language $\left\{a^i b^j c^k \mid j = i + k\right\}$ is context-free.

   (v) _____ The intersection of any three regular languages is regular.

   (vi) _____ The intersection of any regular language with any context-free language is context-free.

   (vii) _____ The intersection of any two context-free languages is context-free.

   (viii) _____ If L is a context-free language over an alphabet with just one symbol, then L is regular.

   (ix) _____ There is a deterministic parser for any context-free grammar.

   (x) _____ The set of strings that your high school algebra teacher would accept as legitimate expressions is a context-free language.

   (xi) _____ Every language accepted by a non-deterministic machine is accepted by some deterministic machine.

   (xii) _____ The problem of whether a given string is generated by a given context-free grammar is decidable.

   (xiii) _____ If G is a context-free grammar, the question of whether $L(G) = \emptyset$ is decidable.

   (xiv) _____ Every language generated by an unambiguous context-free grammar is accepted by some DPDA.

   (xv) _____ The language $\{a^n b^n c^n d^n \mid n \geq 0\}$ is recursive.

   (xvi) _____ The language $\{a^n b^n c^n \mid n \geq 0\}$ is in the class $\mathcal{P}$-TIME.

   (xvii) _____ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a binary numeral.

   (xviii) _____ Every undecidable problem is $\mathcal{NP}$-complete.

   (xix) _____ Every problem that can be mathematically defined has an algorithmic solution.

   (xx) _____ The intersection of two undecidable languages is always undecidable.

   (xxi) _____ Every $\mathcal{NP}$ language is decidable.

(xxii) _____ The intersection of two $\mathcal{NP}$ languages must be $\mathcal{NP}$.

(xxiii) _____ If L_1 and L_2 are $\mathcal{NP}$-complete languages and $L_1 \cap L_2$ is not empty, then $L_1 \cap L_2$ must be $\mathcal{NP}$-complete.

(xxiv) _____ $\mathcal{NC} = \mathcal{P}$.

(xxv) _____ $\mathcal{P} = \mathcal{NP}$.

(xxvi) _____ $\mathcal{NP} = \mathcal{P}$-SPACE

(xxvii) _____ $\mathcal{P}$-SPACE = EXP-TIME

(xxviii) _____ EXP-TIME = EXP-SPACE

(xxix) _____ EXP-TIME = $\mathcal{P}$-TIME.

(xxx) _____ EXP-SPACE = $\mathcal{P}$-SPACE.

(xxxi) _____ The traveling salesman problem (TSP) is $\mathcal{NP}$-complete.

(xxxii) _____ The knapsack problem is $\mathcal{NP}$-complete.

(xxxiii) _____ The language consisting of all satisfiable Boolean expressions is $\mathcal{NP}$-complete.

(xxxiv) _____ The Boolean Circuit Problem is in $\mathcal{P}$.

(xxxv) _____ The Boolean Circuit Problem is in $\mathcal{NC}$.

(xxxvi) _____ If L_1 and L_2 are undecidable langugages, there must be a recursive reduction of L_1 to L_2.

(xxxvii) _____ The language consisting of all strings over $\{a, b\}$ which have more a's than b's is LR(1).

(xxxviii) _____ 2-SAT is $\mathcal{P}$-TIME.

(xxxix) _____ 3-SAT is $\mathcal{P}$-TIME.

(xl) _____ Primality is $\mathcal{P}$-TIME.

(xli) _____ There is a $\mathcal{P}$-TIME reduction of the halting problem to 3-SAT.

(xlii) _____ Every context-free language is in $\mathcal{P}$.

(xliii) _____ Every context-free language is in $\mathcal{NC}$.

(xliv) _____ Addition of binary numerals is in $\mathcal{NC}$.

(xlv) _____ Every context-sensitive language is in $\mathcal{P}$.

(xlvi) _____ Every language generated by a general grammar is recursive.

(xlvii) _____ The problem of whether two given context-free grammars generate the same language is decidable.

(xlviii) _____ The language of all fractions (using base 10 numeration) whose values are less than π is decidable. (A *fraction* is a string. "314/100" is in the language, but "22/7" is not.)

(xlix) _____ There exists a polynomial time algorithm which finds the factors of any positive integer, where the input is given as a unary ("caveman") numeral.

(l) _____ For any two languages L_1 and L_2, if L_1 is undecidable and there is a recursive reduction of L_1 to L_2, then L_2 must be undecidable.

(li) _____ For any two languages L_1 and L_2, if L_2 is undecidable and there is a recursive reduction of L_1 to L_2, then L_1 must be undecidable.

(lii) _____ If P is a mathematical proposition that can be written using a string of length n, and P has a proof, then P must have a proof whose length is $O(2^{2^n})$.

(liii) _____ If L is any $\mathcal{NP}$ language, there must be a $\mathcal{P}$–TIME reduction of L to the partition problem.

(liv) _____ Every bounded function is recursive.

(lv) _____ If L is $\mathcal{NP}$ and also co-$\mathcal{NP}$, then L must be $\mathcal{P}$.

(lvi) _____ Recall that if $\mathcal{L}$ is a class of languages, co-$\mathcal{L}$ is defined to be the class of all languages that are not in $\mathcal{L}$. Let $\mathcal{RE}$ be the class of all recursively enumerable languages. If L is in $\mathcal{RE}$ and also L is in co-$\mathcal{RE}$, then L must be decidable.

(lvii) _____ Every language is enumerable.

(lviii) _____ If a language L is undecidable, then there can be no machine that enumerates L.

(lix) _____ There exists a mathematical proposition that can be neither proved nor disproved.

(lx) _____ There is a non-recursive function which grows faster than any recursive function.

(lxi) _____ There exists a machine that runs forever and outputs the string of decimal digits of π (the well-known ratio of the circumference of a circle to its diameter).

(lxii) _____ For every real number x, there exists a machine that runs forever and outputs the string of decimal digits of x.

(lxiii) _____ **Rush Hour**, the puzzle sold in game stores everywhere, generalized to a board of arbitrary size, is $\mathcal{NP}$–complete.

(lxiv) _____ There is a polynomial time algorithm which determines whether any two regular expressions are equivalent.

(lxv) _____ If two regular expressions are equivalent, there is a polynomial time proof that they are equivalent.