Computer Science 477/677 Fall 2019
University of Nevada, Las Vegas
Answers to First Examination September 18, 2019

1. True or False. [5 points each] Write T if the statement is known to be true, F it is known to be false, and O if it is open, meaning that science has not determined whether it is true or false.

   (a) **T** Any decision tree sorting algorithm must make $\Omega(n \log n)$ comparisons in the worst case.

   (b) **F** Bubblesort takes $O(n \log n)$ expected time on an array of size n in the average case, *i.e.* when items are randomly ordered.

   (c) **T** Mergesort takes $\Theta(n \log n)$ time on an array of size n.

   (d) **O** $\mathcal{P}$–TIME $= \mathcal{NP}$–TIME.

   (e) **T** Quicksort takes $O(n \log n)$ expected time on an array of size n in the average case, *i.e.* when items are randomly ordered.

   (f) **F** There is a polynomial time algorithm for the halting problem.

2. Fill in the blanks. [5 points each blank.]

   (a) Name a divide-and-conquer searching algorithm.

   binary search

   (b) Name two divide-and-conquer sorting algorithms.

   mergesort

   quicksort

3. In each of the following situations, write O, Ω. Θ in the blank.

   (a) $2^n = O(3^n)$

   (b) $\log(2^n) = \Theta(\log(3^n))$

   (c) $n^{1.01} = \Omega(n \log^2 n)$

   (d) $n^{0.1} = \Omega(\log^2 n)$

   (e) $\sqrt{n} = \Omega(\log^3 n)$

   (f) $n2^n = O(3^n)$

   (g) $\sum_{i=1}^{n} i^k = \Theta(n^{k+1})$

4. Give an asymptotic solution to each currence, using O, Ω, or Θ as appropriate. [5 points each]

   (a) $F(n) \leq F(n-1) + \log n$ $\quad F(n) = O(n \log n)$

   (b) $H(n) \leq 2H(n/2) + O(1)$ $\quad H(n) = O(n)$

   (c) $H(n) \geq 2H(n/2) + n^2$ $\quad H(n) = \Omega(n^2)$

   (d) $K(n) = 3K(n/3) + n.$ $\quad K(n) = \Theta(n \log n)$

   (e) $F(n) = F(n - \sqrt{n}) + \sqrt{n}$ $\quad F(n) = \Theta(n)$

   (f) $G(n) \geq 2G(n-1) + 1$ $\quad G(n) = \Omega(2^n)$

   (g) $T(n) = 4T(n/2) + n^2$ $\quad T(n) = \Theta(n^2 \log n)$

5. [10 points] The following (incomplete) C++ code implements which of the sorting algorithms we have discussed?

```
int A[n];
int main()
 {
  for(int i = 0; i < n; i++)
   for(int j = i+1; j < n; j++)
    if(A[j] < A[i]) swap(A[i],A[j]);
  return 1;
 }
```

   selection sort

6. [20 points] Use polyphase mergesort to sort the list below. Show steps.

```
W C S A B J H U N O M R Q T V D Z X Y

W S BHN MQVZ Y
C AJU ORT D X

CW BHNORTYX
AJSU DMQVZ

ACJSUW
BDHMNOQRTVXYZ

ABCDHJMNOQRSTUVWXYZ
```

7. (a) [10 points] How many numbers above 1 and less than 187 are relatively prime to 187?

   187 = 11·17, the product of two primes. The answer is 10·16 - 1 = 159

(b) [10 points] Find the mod 91 inverse of 4. Ans: 23.

(c) [15 points] When I was in the seventh grade, I read a book which dicussed the number 9^{9^9}. The number had never been written down in decimal notation, but what was known was:

the number of digits,

the first so many (I forget how many) digits, and

the last so many digits.

I found that puzzling at the time, but now I know how those things were calculated. For example, I calculate that the numeral for 9^{9^9} has 369693100 digits. I also calculate that the first five digits are 42812, but I am not so sure of that, because of rounding error.

You can do it, too. What is the last digit of the base ten numeral for 9^{9^9}?

Ans: The last three digits are 289.

8. [20 points] Complete the following sentence: **RSA one-way encryption will not be secure if anyone ever . . .**

Can factor integers in polynomial time.