1. True or False. [5 points each] Write T if the statement is known to be true, F it is known to be false, and O if it is open, meaning that science has not determined whether it is true or false.

(a) T Any decision tree sorting algorithm must make \( \Omega(n \log n) \) comparisons in the worst case.

(b) F Bubblesort takes \( O(n \log n) \) expected time on an array of size \( n \) in the average case, i.e. when items are randomly ordered.

(c) T Mergesort takes \( \Theta(n \log n) \) time on an array of size \( n \).

(d) O \( \mathcal{P} - \text{time} = \mathcal{NP} - \text{time} \).

(e) T Quicksort takes \( O(n \log n) \) expected time on an array of size \( n \) in the average case, i.e. when items are randomly ordered.

(f) F There is a polynomial time algorithm for the halting problem.

2. Fill in the blanks. [5 points each blank.]

(a) Name a divide-and-conquer searching algorithm. 
   
   binary search

(b) Name two divide-and-conquer sorting algorithms.
   
   mergesort
   quicksort

3. In each of the following situations, write \( O \), \( \Omega \), \( \Theta \) in the blank.

(a) \( 2^n = O(3^n) \)

(b) \( \log(2^n) = \Theta(\log(3^n)) \)

(c) \( n^{1.01} = \Omega(n \log^2 n) \)

(d) \( n^{0.1} = \Omega(\log^2 n) \)

(e) \( \sqrt{n} = \Omega(\log^3 n) \)

(f) \( n2^n = O(3^n) \)

(g) \( \sum_{i=1}^{n} i^k = \Theta(n^{k+1}) \)
4. Give an asymptotic solution to each recurrence, using \( O \), \( \Omega \), or \( \Theta \) as appropriate. [5 points each]

(a) \( F(n) \leq F(n - 1) + \log n \) \( F(n) = O(n \log n) \)

(b) \( H(n) \leq 2H(n/2) + O(1) \) \( H(n) = O(n) \)

(c) \( H(n) \geq 2H(n/2) + n^2 \) \( H(n) = \Omega(n^2) \)

(d) \( K(n) = 3K(n/3) + n \) \( K(n) = \Theta(n \log n) \)

(e) \( F(n) = F(n - \sqrt{n}) + \sqrt{n} \) \( F(n) = \Theta(n) \)

(f) \( G(n) \geq 2G(n - 1) + 1 \) \( G(n) = \Omega(2^n) \)

(g) \( T(n) = 4T(n/2) + n^2 \) \( T(n) = \Theta(n^2 \log n) \)

5. [10 points] The following (incomplete) C++ code implements which of the sorting algorithms we have discussed?

```cpp
int A[n];
int main()
{
    for(int i = 0; i < n; i++)
        for(int j = i+1; j < n; j++)
    return 1;
}
```

**selection sort**

6. [20 points] Use polyphase mergesort to sort the list below. Show steps.

```
W C S A B J H U N O M R Q T V D Z X Y
W S BHN MQVZ Y
C AJU ORT D X
CW BHNORTYX
AJSU DMQVZ
ACJSUW
BDHMNOQRTVXYZ
ABCDHJINNORSTUWVXYZ
```

7. (a) [10 points] How many numbers above 1 and less than 187 are relatively prime to 187?

\( 187 = 11 \times 17 \), the product of two primes. The answer is 10 \( \cdot \) 16 - 1 = 159
(b) [10 points] Find the mod 91 inverse of 4. Ans: 23.

(c) [15 points] When I was in the seventh grade, I read a book which discussed the number $9^{9^9}$. The number had never been written down in decimal notation, but what was known was:

- the number of digits,
- the first so many (I forget how many) digits, and
- the last so many digits.

I found that puzzling at the time, but now I know how those things were calculated. For example, I calculate that the numeral for $9^{9^9}$ has 369693100 digits. I also calculate that the first five digits are 42812, but I am not so sure of that, because of rounding error.

You can do it, too. What is the last digit of the base ten numeral for $9^{9^9}$?

Ans: The last three digits are 289.

8. [20 points] Complete the following sentence: **RSA one-way encryption will not be secure if anyone ever** . . .

Can factor integers in polynomial time.