University of Nevada, Las Vegas Las Vegas Computer Science 477/677 Fall 2019

Assignment 9 Due Wednesday December 4, 2019

Print out the document, staple, and fill in the answers. Finish by December 4, but do not turn in.

1. Solve each recurrence, expressing the answers using O, Ω, or Θ, whichever is most appropriate.

   (a) $F(n) = 4F(n/2) + n)$

   (b) $F(n) = F(n/2) + \log n$ (Hint: use substitution.)

   (c) $F(n) = F(n-2) + \log n$ (Hint: do not be misled by irrelevancies.)

   (d) $F(n) = F(n - \sqrt{n}) + n$ (Hint: divide by sides by something.)

(e) $F(n) = 3(F(n/3) + F(2n/3)) + n^2$

(f) $F(n) = F(n/2) + F(n/3) + F(n/6) + 1$

2. Explain how to find the median of n items, deterministically, in $O(\log n)$ time using n processors. Can you do it with asymptotically fewer processors, but still in $O(\log n)$ time?

3. Consider a union/find problem where there are n items, and the total number of `find` operations is n and the total number of `union` operations is also n. Assume that you use path compression.

   (a) Is the time complexity $O(n)$? (Hint: No.)

   (b) What is the time complexity?

4. $2n$ items are placed into an open hash table of size n, using a pseudo-random hash function.

   (a) What is the average number of items in a bucket? (Hint: 2.)

   (b) Approximately how many buckets will have no items?

   (c) Approximately how many buckets will have exactly one item?

   (d) We say that a two items *collide* if they have the same hash value. Approximately how many other items does a given item x collide with?

5. You are given an acyclic directed graph $G = (V, E)$.

   (a) Write an algorithm which finds a topological ordering of V.

   (b) Write an algorithm which finds the longest path in G.

(c) Write an algorithm which finds the transitive closure of G.

(d) Write an algorithm which finds the transitive reduction of G.

6. You can only type 80 characters on a line. You are given a sequence of words, $w_1, w_2, \ldots w_n$ of various lengths, which do not fit into one line. You want to construct a paragraph, where each line is as long as possible without exceeding 80 characters. The last line can have any length. No word has length greater than 80, and there must be a space between any two consecutive words. Design a dynamic programming algorithm for this problem. (There is a linear, that is, $O(n)$, time algorithm for this problem.)