**University of Nevada, Las Vegas Computer Science 477/677 Fall 2020**

**Answers to Assignment 2: Due Wednesday September 9, 2020**

**Name:**______________________________________________________________

You are permitted to work in groups, get help from others, read books, and use the internet. Your answers must be written in a pdf file and emailed to the graudate assistant, Tandreana Chua `chuat4@unlv.nevada.edu` , by midnight September 9. Your file must not exceed 5 kilobytes, and must print out to at most 4 pages.

1. Each of these code fragments takes if $O(n \log n)$.time, but not necessarily $\Theta(n \log n)$. Give the asymptotic complexity of each in terms of $n$, using $\Theta$ in each case.

   (a)
   ```
   for(int i = 1; i < n; i++)
    for(int j = 1; j < i; j = 2*j);
     cout << "Hello" << endl;
   ```
   $\int_{x=1}^{n}(lnx)dx = x\ln x - x|_{x=1}^{n} = \Theta(n\log n)$

   (b)
   ```
   for(int i = 1; i < n; i++)
    for(int j = i; j < n; j = 2*j);
     cout << "Hello" << endl;
   ```
   $\int_{x=1}^{n}(\ln n - \ln x)dx = x\ln x - x\ln x + x|_{x=1}^{n} = \Theta(n)$

   (c)
   ```
   for(int i = 1; i < n; i=2*i)
    for(int j = 1; j < i; j++);
     cout << "Hello" << endl;
   ```
   Let $\mathrm{k} = \log_2 \mathrm{i}$; then $2^{\mathrm{k}} = \mathrm{i}$.

   ```
   for(int k = 0; i < log_2 n; k++)
    for(int j = 1; j < 2^k; j++);
     cout << "Hello" << endl;
   ```

   Let $x$ be the continuous analog of k and $y$ the continuos analog of j.

   $\int_{x=0}^{log_2 n}\int_{y=1}^{2^x} dydx = \int_{x=0}^{log_2 n}(2^x - 1)dx = \left.\frac{2^x - x}{\ln 2}\right|_{0}^{\log_2 n} = \frac{2^{\log_2 n} - 1}{\ln 2} = \frac{n-1}{\ln 2} = \Theta(n)$

   (d)
   ```
   for(int i = 1; i < n; i=2*i)
    for(int j = i; j < n; j++);
      cout << "Hello" << endl;
   ```
   Let $\mathrm{k} = \log_2 \mathrm{i}$; then $2^{\mathrm{k}} = \mathrm{i}$.

   ```
   for(int k = 0; i < log_2 n; k++)
    for(int j = 2^k; j < n; j++);
     cout << "Hello" << endl;
   ```

   Let $x$ be the continuous analog of k and $y$ the continuos analog of j.

   $\int_{x=0}^{log_2 n}\int_{y=2^x}^{n} dydx = \int_{x=0}^{log_2 n}(n - 2^x)dx = \left.\left(nx - \frac{2^x}{\ln 2}\right)\right|_{x=0}^{\log_2 n}$

   $= n\log_2 n - \frac{2^{\log_2 n} - 1}{\ln 2} = n\log_2 n - \frac{n-1}{\ln 2} = \Theta(n\log n)$

(e)
```
for(int i = n; i > 1; i=i/2)
 for(int j = i; j > 1; j--);
  cout << "Hello" << endl;
```
Same as (c). $\Theta(n)$

(f)
```
for(int i = n; i > 1; i=i/2)
 for(int j = n; j > i; j--);
  cout << "Hello" << endl;
```
Same as (d). $\Theta(n \log n)$

2. These problems are harder than the ones above. Given the asymptotic complexity of each fragment in terms of $n$, using $\Theta$.

(g)
```
for(int i = 1; i < n; i=2*i)
 for(int j = 1; j < i; j=2*j);
  cout << "Hello" << endl;
```
Hint: Use substitution. Let `m = log n, k = log i, l = log j.`

```
for(int k = 0; k < m; k++)
 for(int l = 0; i < k; l++)
  cout << "Hello" << endl;
```
$\Theta(m^2) = \Theta(\log^2 n)$

(h)
```
for(int i = 2; i < n; i=i*i)
 cout << "Hello" << endl;
```
Hint: Use substitution. Let `m = log n, k = log i.`

Use the fact that $\log(x^y) = y \log x$

```
for(int k = 1; k < m; k=2*k)
 cout << "Hello" << endl;
```
$\Theta(\log m) = \Theta(\log \log n)$

(i)
```
for(int i = 2; i < n; i=i*i)
 for(int j = 1; j < i; j = 2*j)
  cout << "Hello" << endl;
```
Hint: Use substitution. Let `m = log n, k = log i, l = log j.`

```
for(int k = 1; k < m; k=2*k)
 for(int l = 0; l < k; l++)
```
$\Theta(m) = \Theta(\log n)$

(j) `for(int i = n; i > 1; i = log i)`

```
 cout << "Hello" << endl;
```

Hint: The answer is a function you've possibly never heard of. That function is defined on page 136 of the textbook. I will simply tell you the answer, and you will need to remember it.

The function is $\log^* n$, which is defined recursively:

i. $\log^* 1 = 0$

ii. $\log^* n = 1 + \log^*(\log n)$ if $n > 1$ (Logarithms are base 2, of course. Truncate $\log n$ down to an integer.)

Use the substitution $m = \log^* n, k = \log^* i$

```
for(int k = m; k > 0; k--)
```

$\Theta(m) = \Theta(\log^* n)$

What is $\log^* 65536$? Answer: 4.

Let $N$ be the number of baryons in the visible universe. (Neutrons and protons are baryons.) What is $\log^* N$?