Computer Science 477/677 Fall 2020

University of Nevada, Las Vegas Computer Science 477/677 Fall 2020

Answers to Practice for the Final Examination

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The entire practice test is 565 points.

1. True or False. Write "O" if the answer is not known to science at this time. [5 points each]

   (a) **F** Computers are so fast today that complexity theory is only of theoretical, but not practical, interest.

   (b) **T** The inverse Ackermann function, $\alpha(n)$, grows so slowly that, from a practical (as opposed to theoretical) point of view, it might as well be constant.

   (c) **O** If a problem is $\mathcal{NP}$-complete, there is no polynomial time algorithm which solves it.

   (d) **F** Quicksort takes $\Theta(n \log n)$ time on an array of size n.

   (e) **T** Planar graphs are sparse.

   (f) **T** Acyclic graphs are sparse.

   (g) **F** Acyclic directed graphs are sparse.

2. Fill in the blanks. [5 points each blank.]

   (a) If a planar graph $\mathcal{G}$ has 20 edges, then the number of vertices of $\mathcal{G}$ cannot be less than **9**.

   (b) A directed acyclic graph with 5 vertices cannot have more than 10 arcs, and a directed acyclic graph with 6 vertices cannot have more than 15 arcs. A directed acyclic graph with 10 vertices cannot have more than **45** arcs.

   (c) A directed acyclic graph with 20 arcs cannot have fewer than **7** vertices. (You must give the best possible answer, exactly. No partial credit.)

   (d) The height of a binary tree with 50 nodes is at least **5**.

   (e) In **perfect** hashing, there are no collisions.

   (f) If separate chaining is used to resolve collisions in a hash table with n items and n places in the array and if the hash function is pseudo-random, then approximately **8%** of the places will have more than two items. Pick the best answer from among these choices: (0%, 1%, 2%, 4%, 8%, 16%, 32%)

   Hint: approximately 36.8% of the places will have no items.

   (g) $\Omega(n \log n)$.

   (h) **Radix**, or **bucket** sorting is not comparison-based.

   (i) The infix expression $(x + y) * z$ is equivalent to the prefix expression $* + \mathbf{xyz}$ and the postfix expression $\mathbf{xy} + \mathbf{z}*$.

(j) What is the **only** difference between the abstract data types *queue* and *stack*?

In a stack, only the most recently inserted item may be deleted (LIFO), while in a queue, only the least recently inserted item may be deleted (LIFO).

(k) The items stored in a priority queue (that includes stacks, queues, and heaps) represent **unfulfilled obligations.**

(l) Name a divide-and-conquer searching algorithm.

Binary search.

(m) Name two divide-and-conquer sorting algorithms.

Mergesort and quicksort.

(n) The following is pseudo-code for which sorting algorithm we've discussed?

Selection sort.

```
int x[n];
obtain values of x;
for(int i = n-1; i > 0; i--)
 Find the largest element of x[0], ... x[i] and swap it with x[i]
```

(o) The following is pseudo-code for which sorting algorithm we've discussed?

Bubblesort.

```
int x[n];
obtain values of x;
bool finished = false;
for(int i = n-1; i > 0 and not finished; i--)
 {
  finished = true;
  for(int j = 0; j < i; j++)
    if(x[j] > x[j+1])
     {
      swap(x[j],x[j+1]);
      finished = false;
     }
 }
```

3. Give the asymptotic complexity, in terms of `n`, of each of the following code fragments. [10 points each]

(a)
```
for(int i = n; i > 1; i = i/2)
  cout << "hello world" << endl;
```

$\Theta(\log n)$

(b)
```
for(int i = 1; i < n; i++)
  for(int j = 1; j < i; j = 2*j)
    cout << "hello world" << endl;
```

$\Theta(n \log n)$

(c)
```
for(int i = 1; i < n; i++)
  for(int j = i; j < n; j = 2*j)
    cout << "hello world" << endl;
```

$\Theta(n)$

(d)
```
for(int i = 2; i < n; i = i*i)
  cout << "hello world" << endl;
```

$\Theta(\log\log n)$

4. [10 points] Name one problem which is known to be $\mathcal{NP}$-complete. ______________________________

5. Solve the recurrences. Give asymptotic answers in terms of n, using either O, Ω, or Θ, whichever is most appropriate. (10 points each)

(a)	$F(n) = 2F\left(\frac{n}{2}\right) + n$	$\Theta(n\log n)$
(b)	$F(n) \geq 4F\left(\frac{n}{2}\right) + n^2$	$\Theta(n^2\log n)$
(c)	$F(n) = F(n-1) + \frac{n}{4}$	$\Theta(n^2)$
(d)	$F(n) \leq F\left(\frac{n}{2}\right) + F\left(\frac{n}{4}\right) + F\left(\frac{n}{5}\right) + n$	$\Theta(n)$
(e)	$F(n) = F\left(n - \sqrt{n}\right) + n$	$\Theta\left(n^{3/2}\right)$
(f)	$F(n) = F(\log n) + 1$	$\Theta(\log^* n)$

6. [20 points] Use dynamic programming to compute the length of the longest common subsequence of the strings "011011001" and "1010011001."

		1	0	1	0	0	1	1	0	0	1
	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1	1	1	1	1
1	0	1	1	2	2	2	2	2	2	2	2
1	0	1	1	2	2	2	3	3	3	3	3
0	0	1	2	2	3	3	3	3	4	4	4
1	0	1	2	3	3	3	4	4	4	4	5
1	0	1	2	3	3	3	4	5	5	5	5
0	0	1	2	3	4	4	4	5	6	6	6
0	0	1	2	3	4	5	5	5	6	7	7
1	0	1	2	3	4	5	6	6	6	7	8

The length of the longest common subsequence is 8.

7. [20 points] Use dynamic programming to compute the Levenshtein distance between the strings “abcdabc” and “bdacbcd.”

		a	b	c	d	a	b	c
	0	1	2	3	4	5	6	7
b	1	1	1	2	3	4	5	6
d	2	2	2	2	2	3	4	5
a	3	2	3	3	3	2	3	4
c	4	3	3	3	4	3	3	3
b	5	4	3	4	4	4	3	4
c	6	5	4	3	4	5	4	3
d	7	6	5	4	3	4	5	4

The Levenshtein distance between those two strings is 4.

8. [20 points] Design a dynamic programming to compute the maximum sum of any contiguous subsequence of a given sequence of numbers. For example, if the given sequence is $2, 1, -4, 6, = 3, 7, -1.2. - 2, 1$ that sum is $6 + (-3) + 7 + (-1) + 2 = 11$. (There is an $O(n)$-time algorithm.)

Let $x_1, x_2, \ldots x_n$ be the sequence. We construct two arrays. Let `A[i]` be the largest sum of any contiguous subsequence of $x_1, x_2, \ldots x_i$, and let `B[i]` be the largest sum of any contiguous subsequence of $x_1, x_2, \ldots x_i$ which includes x_i. Write `X[i]` for x_i.

```
A[0] = 0;
B[1] = X[1]
A[1] = max{0.B[1]}
for(i = 2 to n)
 {
  B[i] = X[i} + max{0,B[i-1]}
  A[i] = max{A[i-1],B[i]}
 }
write A[n]
```

9. Solve each of the following recurrences, giving the answer in terms of O, Θ, or Ω, whichever is most appropriate [10 points each].

(a) $T(n) < T(n-2) + n^2$ $\quad T(n) = O(n^3)$

(b) $F(n) \geq F(\sqrt{n}) + \lg n$ $\quad F(n) = \Omega(\log n)$

(c) $G(n) \geq G(n-1) + n$ $\quad G(n) = \Omega(n^2)$

(d) $F(n) = 4F(n/2) + n^2$ $\quad F(n) = \Theta(n^2)$

(e) $H(n) \leq 2H(\sqrt{n}) + O(\log n)$ $\quad H(n) = O(\log n \log\log n)$

(f) $K(n) = K(n - \sqrt{n}) + 1$ $\quad K(n) = \Theta(\sqrt{n})$

(g) $F(n) = 4F\left(\frac{3n}{4}\right) + n^5$ $\quad F(n) = \Theta(n^5)$

10. Find the asymptotic complexity, in terms of n, for each of these fragments, expressing the answers using O, Θ, or Ω, whichever is most appropriate.

   (a)
   ```
   for(i = 0; i < n; i = i+1);
   cout << "Hi!" << endl;
   ```
   $\Theta(n)$

   (b)
   ```
   for(i = 1; i < n; i = 2*i);
   cout << "Hi!" << endl;
   ```
   $\Theta(\log n)$

   (c)
   ```
   for(i = 2; i < n; i = i*i);
   cout << "Hi!" << endl;
   ```
   $\Theta(\log \log n)$

   (d) The following code models the first phase of heapsort.
   ```
   for(int i = n; i > 0; i--)
    for(int j = i; 2*j <= n; j = 2*j)
     cout << "swap" << endl;
   ```
   $\Theta(n)$

   (e) The following code models the second phase of heapsort.
   ```
   for(int i = n; i > 0; i--}
    {
     cout << "swap" << endl;
     for(int j = 1; 2*j <= i; j = 2*j)
      cout << "swap" << endl;
    }
   ```
   $\Theta(n \log n)$

   (f) The following code models insertion of n items into an AVL tree.
   ```
   for(int i = 1; i < n; i++)
    for(int j = n; j > 0; j = j/2)
      cout << "check AVL property and possibly rotate" << endl;
   ```
   $\Theta(n \log n)$

11. Solve each of the following recurrences, expressing the answers using O, Θ, or Ω, whichever is most appropriate. [10 points each]

   (a) $F(n) = F(n/2) + 1$ $\quad F(n) = \Theta(\log n)$

   (b) $F(n) = F(n-1) + O(\log n)$

   (c) $F(n) = F\left(\frac{n}{2}\right) + 2F\left(\frac{n}{4}\right) + n$ $\quad F(n) = \Theta(n \log n)$

   (d) $F(n) = F\left(\frac{3n}{5}\right) + F\left(\frac{4n}{5}\right) + n^2$ $\quad F(n) = \Theta(n^2 \log n)$

   (e) $F(n) = F(n-2) + n$ $\quad F(n) = \Theta(n^2)$

12. Use Huffman's algorithm to construct an optimal prefix code for the alphabet $\{A, B, C, D, E, F\}$ where the frequencies of the symbols are given by the following table.

A	2
B	8
C	9
D	3
E	7
F	5

13. [10 points] Write pseudo-code for binary search.

Assume that `{X[i]}` for $0 \leq$ i $<$ n is an ordered array. Let x be the sought value.

```
lo = 0
hi = n
while(lo+1 < hi)
 {
  mid = (lo+hi)/2
  if(x < X[mid]) hi = mid
  else lo = mid
 }
if(x = X[hi]) write "x is found in position hi"
else write "x is not in the array"
```

14. Find the asymptotic complexity, in terms of n, for each of these fragments, expressing the answers using O, Θ, or Ω, whichever is most appropriate. [10 points each]

(a)
```
for(int i = 1; i*i < n; i++)
  cout << "Hi!" << endl;
```
$\Theta(\sqrt{n})$

(b)
```
for(int i = n; i > 1; i = sqrt(i));
  cout << "Hi!" << endl;
```
$\Theta(\log\log n)$

Find the asymptotic time complexity, in terms of n, for each of these functions, expressing the answers using O, Θ, or Ω, whichever is most appropriate. [10 points each]

(a)
```
int f(int n)
  {
   if (n < 2) return 1;
   else return f(n-1)+f(n-1);
  }
```
$\Theta(2^n)$

(b)
```
void hello(int n)
   {
```

```
    if(n >= 1)
     {
      for(int i = 1; i < n; i++)
       cout << "Hello!" << endl;
      hello(n/2);
      hello(n/2);
     }
   }
```

$\Theta(n \log n)$

15. [20 points] Define the Collatz function as follows:

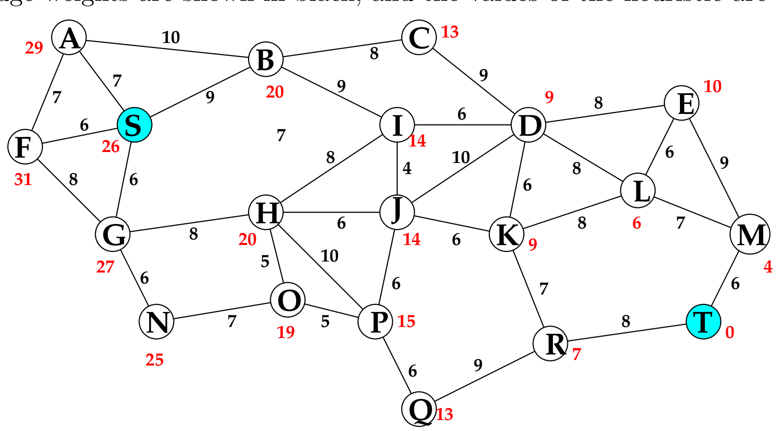
```
int collatz(int n)
  {
    assert(n > 0);
    if(n == 1) return 0;
    else if (n%2) return collatz(3*n+1); // n is odd, greater than 1
    else return collatz(n/2); // n is even
   }
```

Write pseudo-code for a memoization algorithm which prints collatz(n) for all n from 1 to 1000.

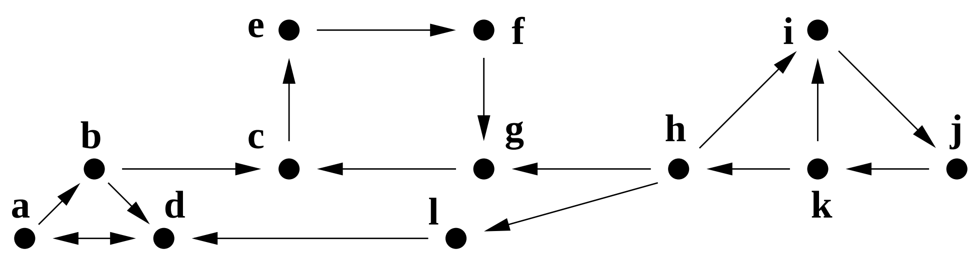
16. [20 points] Give pseudocode for a recursive algorithm which computes the median of the union of two sorted lists in logarithmic time.

17. [20 points] Describe a randomized algorithm which finds the k^{th} smallest element of an unsorted list of n distinct numbers, for a given $k \leq n$, in $O(n)$ expected time. (By "distinct," I mean that no two numbers in the list are equal.)

18. [20 points] Walk through the A^* algorithm for the following weighted graph to find the shortest path from S to T. Edge weights are shown in black, and the values of the heuristic are shown in red.



19. [20 points] Circle the strong components of the directed graph.



20. [20 points] Give pseudocode for the Bellman-Ford algorithm.

21. [20 points] Give pseudocode for the Floyd-Warshall algorithm.

```
for(int i = 0; i < n; i++)
 for(int j = 0; j < n; j++)
  dist[i,j] = infinity;
for(int i = 0; i < n; i++)
 dist[i,i] = 0;
for(int j = 0; j < n; j++)
 for(int i = 0; i < n; i++)
  for(int k = 0; i < n; i++)
   {
    temp = dist[i,j]+dist[j,k];
    if(tenp < dist[i,k])
     {
      dist[i,k] = temp;
      back[i,k] = i;
     }
   }
```

22. [20 points] Show the minimum spanning tree of the following weighted graph.