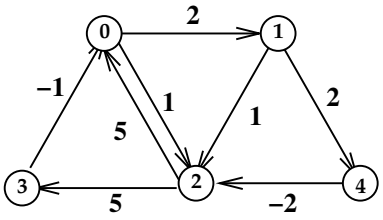
Floyd Warshall Algorithm

N is the number of vertices in a directed graph G. The Floyd Warshall algorithm solves the all pairs shortest path problem. The name of each vertex is an integer in the range $0 \ldots N-1$.

We let $W[i][j]$ be the weight of the arc from i to j. We let the default value of the array be "infinity," that is, we say $W[i,j] = \infty$ if there is no arc from i to j. C++ actually has no variable called "infinity." How do we handle that? Here is C++ code.

```
int V[N][N];
int back[N][N];

int main()
 {
  for(int i = 0; i < N; i++)
   for(int j = 0; j < N; j++)
    {
     V[i][j] = W[i][j];
     back[i][j] = i;
    }
  for(int i = 0; i < N; i++)
   V[i][i] = 0;
  for(int j = 0; j < N; j++)
   for(int i = 0; i < N; i++)
    for(int k = 0; k < N; k++)
     {
      int temp = V[i][j]+V[j][k];
      if(temp < V[i][k])
       {
        V[i][k] = temp;
back[i][k] = back[j][k];
       }
     }
  return 1;
 }
```



W

	0	1	2	3	4
0		2	1	∞	∞
1	∞		1	∞	2
2	5	∞		5	∞
3	−1	∞	∞		∞
4	∞	∞	−2	∞	

V

	0	1	2	3	4
0	0	2	1	6	4
1	4	0	0	5	2
2	4	6	0	5	8
3	−1	1	0	0	3
4	2	4	−2	3	0

back

	0	1	2	3	4
0		0	0	2	1
1	3		4	2	1
2	3	0		2	1
3	3	0	0		1
4	3	0	4	2	

Here is an example for $N = 5$. The time complexity of this algorithm is $\Theta(N^3)$. What is the loop invariant of the outer loop? That loop invariant is important for proving the correctness of the algorithm.