

Topics that might be on the second examination on October 26 2022

A few time complexity and recurrence problems, since not everyone has mastered them.

Be able to write pseudocode:

1. Floyd Warshall algorithm.  $W$  are the arc weights.

For all  $i$  and all  $j$   $V[i,j] = W[i,j]$ ,  $back[j] = i$

```
For all j
  For all i
    For all k
      { temp = V[i,j]+V[j,k]
        if(temp < V[i,k])
          V[i,k] = temp
      }
back[i,k] = back[j,k]
}
```

2. Bellman Ford algorithm.  $0 = source$ .

```
For all  $i$   $V[i] = W[0,i]$ ,  $back[i] = 0$ 
Repeat the following until there are no more changes
{
  for all arcs  $(i,j)$ 
    { temp =  $V[i]+W[i,j]$ 
      if(temp <  $V[j]$ )
         $V[j] = temp$ 
    }
back[j] =  $i$ 
}
```

Be able to step through Dijkstra's algorithm for a small graph.

Johnson's algorithm.

Given a figure showing a weighted directed graph with some negative weights,  
Update the weights on edges so that there are no negative weights.

What is the worst-case time complexity for each of those four algorithms?

Johnson's  $O(nm \log n)$  Floyd Warshal  $\Theta(n^3)$  Bellman Ford  $O(nm)$   
Dijkstra  $O(m \log n)$

Dynamic Programming, possibly with memoization.

Example problem: maximum sum contiguous subsequence. Given a sequence of numbers, each either positive or negative, find a contiguous subsequence of maximum total. As an example if the input sequence is 3,-4,8,-2,3,5,-1,6,-5 the output sequence is 8,-2,3,5,-1,6. There are several algorithms for this problem.

1. Dumb Brute Force:  $O(n^3)$  time.
2. Slightly Smarter Brute Force:  $O(n^2)$  time.
3. Divide and Conquer:  $O(\log n)$  time.
4. Dynamic Programming:  $O(n)$  time.

BFPRT (median of medians) algorithm for Selection.

Explain how to obtain the recurrence for the time complexity, that is  $T(n) \leq T(n/5) + T(7n/10) + n$ .

It takes  $O(1)$  time to find the median of all blocks of 5. That's  $\Theta(n)$ . Finding the median of those medians, to use as a pivot, takes  $T(n/5)$  time. Using the pivot, we partition the set into two pieces. Each piece has at least  $3n/10$  items, hence each piece has no more than  $7n/10$  items.

Find components of an undirected graph using union/find.

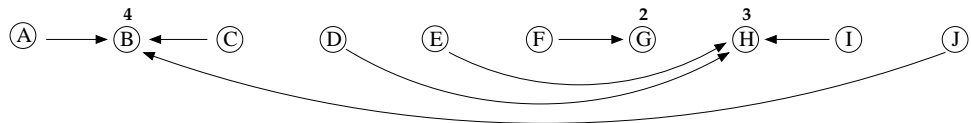
Here is an example. The graph is given to us as a list of edges. Let the vertices be Roman letters in the range A .. J. Let the list of edges be:

Work this example.

Each time you see an edge, you do find for both vertices, then union for both leaders if they are different.

The final diagram is shown below.

- {A, B}
- {E, H}
- {F, G}
- {C, B}
- {D, E}
- {A, J}
- {D, I}
- {D, H}
- {B, J}
- {C, J}
- {E, H}



Heapsort.  $O(n \log n)$  time. A sped up version of selection sort.