University of Nevada, Las Vegas Computer Science 477/677 Fall 2023

Complex Number Assignment Part 1

Name:____

You are permitted to work in groups, get help from others, read books, and use the internet. Your answers must be written in a pdf file and uploaded to canvas, by midnight September 9th. Your file must not be unnecessarily long. If you have any questions, or you are having trouble uploading the assignment you may email the grader, Sepideh Farivar, at `farivar@unlv.nevada.edu`. You may also send me email to ask questions.

The Complex Number System

As you know, there is a square root of every positive real number, but there is no real number which is the square root of any negative real number. In particular, $\sqrt{-1}$ does not exist in the real number system, $\mathbb{R}$. The complex system $\mathbb{C}$ is the algebraic extension of $\mathbb{R}$ obtained by adjoining the imaginary[1] unit $\mathbf{i}$, where $\mathbf{i}^2 = -1$.

In the problems below, if I ask for $a + b\mathbf{i}$ form, I mean that you must not use transcendental functions such as sin and cos.

1. Perform the indicated operations. All answers must be in the $a + b\mathbf{i}$ form.[2] and reduced to the lowest terms.
   (a) $(2 + 3\mathbf{i}) - (1 - 2\mathbf{i})$
   (b) $(2\mathbf{i}) - (4 + 3\mathbf{i})$
   (c) $\dfrac{4 + 3\mathbf{i}}{1 + 2\mathbf{i}}$
2. Find the modulus and argument of each of these complex numbers.
   (a) $\cos(-\frac{\pi}{4}) + \mathbf{i}\sin(-\frac{\pi}{4})$
   (b) $1 + \mathbf{i}$
   (c) $e^{1+\frac{\pi}{3}\mathbf{i}}$
3. Write the six 6^{th} roots of unity in $a + b\mathbf{i}$ form.
4. Write the five 5^{th} roots of unity in Polar form.
5. Write the two square roots of $1 + \mathbf{i}$ in $a + b\mathbf{i}$ form.
6. Write the eight 8^{th} roots of unity in $a + b\mathbf{i}$ form.
7. Write $e^{\frac{\pi}{8}\mathbf{i}}$ in $a + b\mathbf{i}$ form. Hint: you will need to compute complex square roots.

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[1] The word "imaginary" gives the false impression that the imaginary numbers are like kobolds or leprechauns. But they do exist, in the same sense that integers or real numbers exist – as abstract, not physical, objects.

[2] Strictly speaking, $\frac{1+\mathbf{i}}{2}$ is not in the $a + b\mathbf{i}$ form, but the equivalent expression $\frac{1}{2} + \frac{1}{2}\mathbf{i}$ is. However, I will allow a complex number to be written as a fraction where the numerator is complex and the denominator is positive real. Thus, you can write $\frac{1+\mathbf{i}}{2}$, but not $\frac{1+\mathbf{i}}{-2}$ or $\frac{1+\mathbf{i}}{2\mathbf{i}}$. Those should be written $\frac{-1-\mathbf{i}}{2}$ and $\frac{1-\mathbf{i}}{2}$, respectively.