University of Nevada, Las Vegas Computer Science 477/677 Fall 2023

Assignment 7: Due Sunday November 19, 2023, 11:59 PM

Name:_____

You are permitted to work in groups, get help from others, read books, and use the internet. Turn the assignment in to Canvas, following the instructions given to you by Sabrina Wallace.

1. Fill in the blanks.
   (a) In _____ hashing, no two data have the same hash value.
   (b) Using _____ coding, the codons for the different symbols of a message may be written consecutively without spaces.
2. Give the asymptotic time complexity in terms of n, using Θ, O, or Ω, whichever is most appropriate.
   (a) $F(n) \geq F(n - \sqrt{n}) + n^2$
   (b) $H(n) < H(n/3) + H(n/4) + 2H(n/5) + n$
   (c) $G(n) = 3(G(2n/3) + G(n/3)) + 5n^2$
   (d) $F(n) = 2F(n-1) + 1$
3. For each of these recursive subprograms, write a recurrence for the time complexity, then solve that recurrence.
   (a)
```
void george(int n)
 {
  if(n > 0)
   {
    for(int i = 0; i < n; i++) cout << "hello" << endl;
    george(n/2); george(n/3); george(n/6);}}
   }
 }
```
   (b)
```
void martha(int n)
 {
  if (n > 1)
   {
    martha(n-1);
    martha(n-2);
```

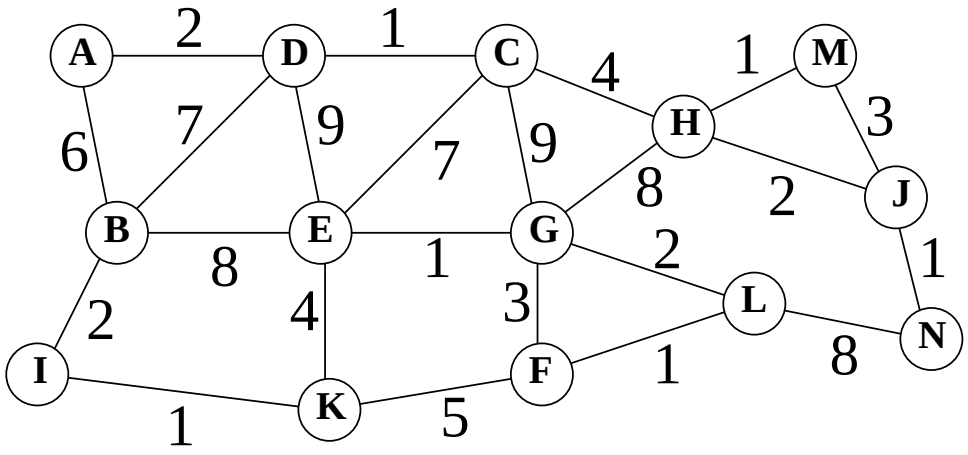
```
 }
else cout << "hello world.";
}
```

Hint: Look at problem 0.3 on page 9 of your textbook.

4. A hash table with of size $m = 200$ is used to store 400 data items, using a pseudo-random hash function. The average number of items in a cell ("bucket") is clearly $\frac{400}{200} = 2$, but there could be empty cells. What is the expected number of empty cells? Approximate to the nearest integer. Hint: You may need to look in a statistics textbook, or on the internet, to figure this out.

5. Let $\sigma = x_1, x_2, \ldots x_n$ be a sequence of numbers with both positive and negative terms. Write an $O(n)$ time dynamic program which finds maximum sum of any contiguous subsequence of σ. For example, if the sequence is $-1, 4, -3, 2, 7, -5, 3, 4, -6, 8, -1$ then the answer is $4 - 3 + 2 + 7 - 5 + 3 + 4 - 6 + 8 = 14$. vskip 0.4in

6. Walk through Kruskal's algorithm to find the minimum spanning tree of the weighted graph shown below. Show the evolution of the union/find structure at several intermediate steps. Whenever there is choice between two edges of equal weight, choose the edge which has the alphabetically largest vertex. Whenever there is a union of two trees of equal weight, choose the alphabetically larger root to be the root of the combined tree. Indicate path compression when it occurs.



7. Insert the letters B, A, Y, H, P, D into an empty treap, where the "random" keys are given in the following table. Show the treap after each insertion and indicate all rotations.

A	8
B	4
D	10
H	2
P	5
Y	9

8. Find an optimal Huffman code on the alphabet A,B,C,D,E,F where frequencies are given in the following table.

A	8
B	6
C	3
D	11
E	14
F	2

9. A 3-dimensional $9 \times 7 \times 10$ rectangular array A is stored in main memory in row major order, and its base address is 8192. Each item of A takes one word of main memory, that is, one addressed location. Find the address, in main memory, of A[4][5][2].

10. You are trying to construct a cuckoo hash table of size 8, where each of the 8 names listed below has the two possible hash values indicated in the array. Using Hall's marriage theorem, prove that you will fail to construct the table.

	h1	h2
Ann	1	0
Bob	4	2
Cal	0	7
Dan	1	6
Eve	1	0
Fay	6	2
Gus	5	3

11. Explain how to implement a sparse one-dimensional array using a search structure.

12. Perform these calculations using complex numbers.

(a) What is the modulus of $\dfrac{-11+2\,\mathbf{i}}{5}$?

(b) Write $\dfrac{-4+3\,\mathbf{i}}{2+\mathbf{i}}$ in $a+b\,\mathbf{i}$ form.

(c) Write the 8^{th} roots of unity in $a+b\,\mathbf{i}$ form.

(d) Write the square roots of $-1+\mathbf{i}$ in $a+b\,\mathbf{i}$ form.

Write the principle 9^{th} root of unity in polar notation.

13. In this problem, assume that each arithmetic operation takes constant time, and that memos can be stored or fetched in constant time.

The function F is computed by the following dynamic program.

```
F(0) = 0
for i from 1 to n
  F(i) = F(i/2) + F((i-1)/2) + i*i
```

(a) What is the asymptotic complexity of $F(n)$?

(b) What is the asymptotic time complexity of the above dynamic program?

(c) Write a recursive program which computes $F(n)$. What is its time complexity?

What is the time complexity of computing $F(n)$ using memoization?