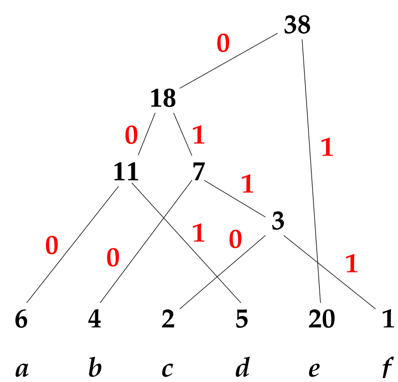
University of Nevada, Las Vegas Computer Science 477/677 Fall 2023

Answers to Assignment 4: Due Friday October 13 2023

1. Fill in the blanks. No shortest path algorithm works for a weighted directed graph which has a **negative cycle**.

2. Use Huffman's algorithm to find an optimal prefix code for the alphabet $\{a, b, c, d, e, f\}$ where frequency of each symbol is given in the following array.

a	6	000
b	4	010
c	2	0110
d	5	001
e	2	1
f	1	00111



It is not necessary to sort the symbols by frequency, but you must maintain a data structure that enables finding the two packages of smallest weight quickly. (A package is a subtree that does not yet have a parent.) Initially sorting the symbols, then placing the packages in an array in the order they are formed, enables the remainder of the algorithm to run in linear (that is, $O(n)$) time.

3. What is the asymptotic time complexity of the function `george(n)` given below? Hint: The answer is one of the following: $\Theta(\log n)$, $\Theta(n)$, $\Theta(n \log n)$, $\Theta(n^2)$.

```
void george(int n)
 {
  for(int i = 0; i < n; i++)
   cout << "Hello world" << end;
  if(n > 1)
   {
    george(n/2);
    george(n/2);
   }
 }
```

Answer: $\Theta(n \log n)$, just like mergesort.

4. Write pseudocode for the Floyd-Warshall algorithm on a weighted directed graph with n vertices. Let the vertices be $x[1], \ldots x[n]$. Let $w[i, j]$ be the given weight of the arc from $x[i]$ to $x[j]$, which is ∞ if there is no such arc. The algorithm should output arrays v and b, where $v[i, j]$ is the minimum weight of any path from $x[i]$ to $x[j]$, and $b[i, j]$ is the backpointer of that path.

```
for all i and j from 1 to n
  v[i,j] = w[i,j]
  b[i,j] = i
```

```
for all i from 1 to n
  v[i,i] = 0;
for all j from 1 to n
  for all i from 1 to n
    for all k from 1 to n
      if(v[i,j] + v[j,k] < v[i,k])
        v[i,k] = v[i,j] + v[j,k]
        back[i,k] = back[j,k]
```

The time complexity of the Floyd–Warshall algorithm is $\Theta(n^3)$.

5. Write pseudocode for the Bellman-Ford algorithm on a weighted directed graph with n vertices and m edges. Let the vertices be $x[1], \ldots x[n]$, let $x[1]$ be the source vertex. and let the arcs (directed edges) be $e[1], \ldots e[m]$, where $e[k]$ is an arc from a vertex $s[k]$ to a vertex $t[k]$, and $e[k]$ has weight $w[k]$. The output consists of values $v[1], \ldots v[n]$, where $v[i]$ is the least weight of any path from $x[1]$ to $x[i]$, as well as backpointers $b[2], \ldots b[n]$. Your code should include the shortcut which terminates the execution when the correct values of the $v[i]$ have been computed.

```
for all i from 2 to n
  v[i] = infinity
v[1] = 0;
finished = false;
while(not finished)
  finished = true;
  for all k from 1 to m // iterate through all arcs
    s = s[k]
    t = t[k]
    w = w[k]
    if(v[s] + w[s,t] < v[t])
      v[t] = v[s] + w[s,t] // update the shortest path from 1 to t by
                           // concatenating the shortest path from 1 to s
                           // with the arc from s to t
      finished = false; // a change has been made, so we may not be done.
// if finished is still true, we're done.
```

6. Find a dynamic programming algorithm for the following coin-row problem. There is a row of n coins, each of which has some value, and the algorithm needs to find a set of coins of maximum total value, such that there are no three consecutive coins in the set.

Let $X[i]$ be the value of the i^{th} coin, and let $M[i]$ be the maximum value if only the first i coins are available. The following program computes $M[i]$ for $i = 0, \ldots n$.
$M[0] = 0$
$M[1] = X[1]$
$M[2] = X[1] + X[2]$, or $M[1] + X[2]$
$M[i] = \max\{M[i-1], X[i] + M[i-2], X[i] + X[i-1] + M[i-3]\}$ for $i \geq 3$
The best set of coins has value $M[n]$

7. Work problems 1 and 5 of the complex number assignment.

1. Perform the indicated operations. All answers must be in the $a + b\mathbf{i}$ form.[1] and reduced to the lowest terms.

   (a) $(2 + 3\mathbf{i}) - (1 - 2\mathbf{i})$

   Ans: $1 + \mathbf{i}$

   (b) $(2\mathbf{i}) - (4 + 3\mathbf{i})$

   Ans: $-4 - \mathbf{i}$

   (c) $\dfrac{4 + 3\mathbf{i}}{1 + 2\mathbf{i}}$

   Ans: Multiply both numerator and denominator by the conjugate of the denominator.

$$\frac{4 + 3\mathbf{i}}{1 + 2\mathbf{i}} = \frac{(4 + 3\mathbf{i})(1 - 2\mathbf{i})}{(1 + 2\mathbf{i})(1 - 2\mathbf{i})} = \frac{10 - 5\mathbf{i}}{5} = 2 - \mathbf{i}$$

5. Find both square roots of $1 + \mathbf{i}$.

   $a = 1$ and $b = 1$. By the fomula given in the handout

$$\begin{aligned} \sqrt{1 + \mathbf{i}} &= \pm\left(\sqrt{\frac{\sqrt{a^2 + b^2} + a}{2}} + \sqrt{\frac{\sqrt{a^2 + b^2} - a}{2}}\,\mathbf{i}\right) \\ &= \pm\left(\sqrt{\frac{\sqrt{2} + 1}{2}} + \sqrt{\frac{\sqrt{2} - 1}{2}}\,\mathbf{i}\right) \end{aligned}$$

[1] Strictly speaking, $\frac{1+\mathbf{i}}{2}$ is not in the $a + b\mathbf{i}$ form, but the equivalent expression $\frac{1}{2} + \frac{1}{2}\mathbf{i}$ is. However, I will allow a complex number to be written as a fraction where the numerator is complex and the denominator is positive real. Thus, you can write $\frac{1+\mathbf{i}}{2}$, but not $\frac{1+\mathbf{i}}{-2}$ or $\frac{1+\mathbf{i}}{2\mathbf{i}}$. Those should be written $\frac{-1-\mathbf{i}}{2}$ and $\frac{1-\mathbf{i}}{2}$, respectively.