University of Nevada, Las Vegas Computer Science 477/677 Fall 2023

Answers to Assignment 5: Due Sunday October 22, 2023

Name:____

You are permitted to work in groups, get help from others, read books, and use the internet. Turn the assignment in to Canvas, following the instructions given to you by Sabrina Wallace.

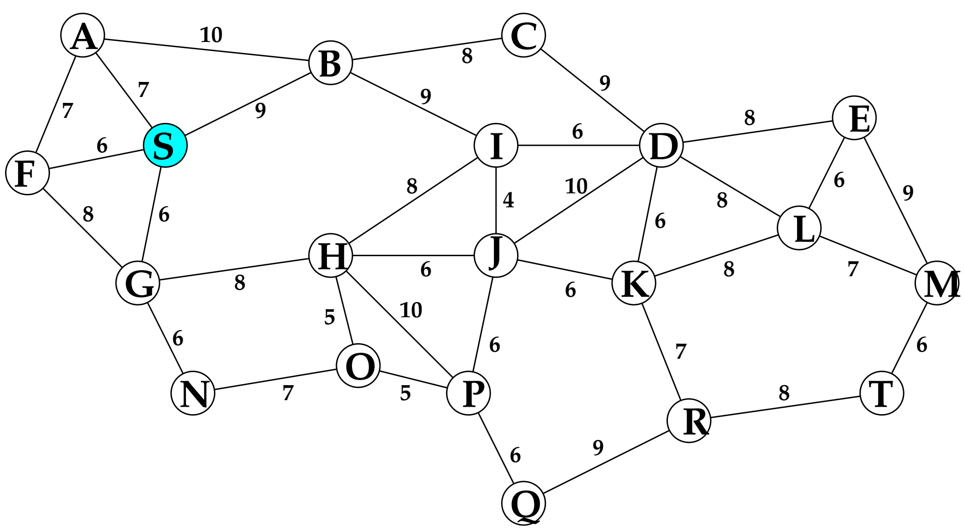
1. Walk through Dijkstra's algorithm for the single source minpath problem for the directed graph illustrated below. Instead of numbering the vertices 0 through 19, I have assigned them letters from A to T. The source vertex is S. As I emailed you from Canvas, each edge is two arcs, one in each direction.

   After each iteration of the main loop, show

   1. The array dist, where dist[x] is the smallest length of any path found so far from S to x. (Initially, dist[x] = ∞ for most x.)

   2. The array back, where back[x] is the next-to-the last vertex on the path of smallest weight found so far from S to x.

   3 The contents of heap. Do not try to show the structure of the heap, simply list its members.



I gave the complete answer after each step. Some steps, such as the last two, did not cause any change in the arrays, but I put them in anyway. I would certainly not expect you do go to so much trouble on a homework assignment, much less an exam. You should make due with far fewer copies of the matrix, crossing out values as necessary. Do not erase the crossed out entries.

	S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	
dist	0	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	{S}
back	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	

	S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	
dist	0	7	9	∞	∞	∞	6	6	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	{F,G,A,B}
back	-	S	S	-	-	-	S	S	-	-	-	-	-	-	-	-	-	-	-	-	

	S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	
dist	0	7	9	∞	∞	∞	6	6	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	{G,A,B}
back	-	S	S	-	-	-	S	S	-	-	-	-	-	-	-	-	-	-	-	-	

	S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	
dist	0	7	9	∞	∞	∞	6	6	14	∞	∞	∞	∞	∞	12	∞	∞	∞	∞	∞	{A,B,N,H}
back	-	S	S	-	-	-	S	S	G	-	-	-	-	-	G	-	-	-	-	-	

	S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	
dist	0	7	9	∞	∞	∞	6	6	14	∞	∞	∞	∞	∞	12	∞	∞	∞	∞	∞	{B,N,H}
back	-	S	S	-	-	-	S	S	G	-	-	-	-	-	G	-	-	-	-	-	

	S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	
dist	0	7	9	17	∞	∞	6	6	14	18	∞	∞	∞	∞	12	∞	∞	∞	∞	∞	{N,H,C,I}
back	-	S	S	B	-	-	S	S	G	B	-	-	-	-	G	-	-	-	-	-	

	S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	
dist	0	7	9	17	∞	∞	6	6	14	18	∞	∞	∞	∞	12	19	∞	∞	∞	∞	{H,C,I,O}
back	-	S	S	B	-	-	S	S	G	B	-	-	-	-	G	N	-	-	-	-	

	S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	
dist	0	7	9	17	∞	∞	6	6	14	18	20	∞	∞	∞	12	19	24	∞	∞	∞	{C,I,O,J,P}
back	-	S	S	B	-	-	S	S	G	B	H	-	-	-	G	N	H	-	-	-	

	S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	
dist	0	7	9	17	26	∞	6	6	14	18	20	∞	∞	∞	12	19	24	∞	∞	∞	{I,O,J,P,D}
back	-	S	S	B	C	-	S	S	G	B	H	-	-	-	G	N	H	-	-	-	

	S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	
dist	0	7	9	17	24	∞	6	6	14	18	20	∞	∞	∞	12	19	24	∞	∞	∞	{O,J,P,D}
back	-	S	S	B	I	-	S	S	G	B	H	-	-	-	G	N	H	-	-	-	

	S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	
dist	0	7	9	17	24	∞	6	6	14	18	20	∞	∞	∞	12	19	24	∞	∞	∞	{J,P,D}
back	-	S	S	B	I	-	S	S	G	B	H	-	-	-	G	N	H	-	-	-	

	S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	
dist	0	7	9	17	24	∞	6	6	14	18	20	26	∞	∞	12	19	24	∞	∞	∞	{P,D,K}
back	-	S	S	B	I	-	S	S	G	B	H	J	-	-	G	N	H	-	-	-	

	S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	
dist	0	7	9	17	24	∞	6	6	14	18	20	26	∞	∞	12	19	24	30	∞	∞	{D,K,Q}
back	-	S	S	B	I	-	S	S	G	B	H	J	-	-	G	N	H	P	-	-	

	S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	
dist	0	7	9	17	24	32	6	6	14	18	20	26	32	∞	12	19	24	30	∞	∞	{K,Q,E,L}
back	-	S	S	B	I	D	S	S	G	B	H	J	D	-	G	N	H	P	-	-	

	S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	
dist	0	7	9	17	24	32	6	6	14	18	20	26	32	∞	12	19	24	30	33	∞	{Q,E,L,R}
back	-	S	S	B	I	D	S	S	G	B	H	J	D	-	G	N	H	P	K	-	

	S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	
dist	0	7	9	17	24	32	6	6	14	18	20	26	32	∞	12	19	24	30	33	∞	{E,L,R}
back	-	S	S	B	I	D	S	S	G	B	H	J	D	-	G	N	H	P	K	-	

	S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	
dist	0	7	9	17	24	32	6	6	14	18	20	26	32	41	12	19	24	30	33	∞	{L,R,M}
back	-	S	S	B	I	D	S	S	G	B	H	J	D	E	G	N	H	P	K	-	

	S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	
dist	0	7	9	17	24	32	6	6	14	18	20	26	32	39	12	19	24	30	33	∞	{R,M}
back	-	S	S	B	I	D	S	S	G	B	H	J	D	L	G	N	H	P	K	-	

	S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	
dist	0	7	9	17	24	32	6	6	14	18	20	26	32	39	12	19	24	30	33	41	{M,T}
back	-	S	S	B	I	D	S	S	G	B	H	J	D	L	G	N	H	P	K	R	

	S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	
dist	0	7	9	17	24	32	6	6	14	18	20	26	32	39	12	19	24	30	33	41	{T}
back	-	S	S	B	I	D	S	S	G	B	H	J	D	L	G	N	H	P	K	R	

	S	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	T	
dist	0	7	9	17	24	32	6	6	14	18	20	26	32	39	12	19	24	30	33	41	$\emptyset$
back	-	S	S	B	I	D	S	S	G	B	H	J	D	L	G	N	H	P	K	R	

Work problems 2, 3, 4, 6 and 7 of the complex number assignment, cmplxhw1.pdf.

2. Find the modulus and argument of each of these complex numbers.

   (a) $\cos(-\frac{\pi}{4}) + \imath \sin(-\frac{\pi}{4})$

   The modulus is 1, the argument is 225^o, or $\frac{5\pi}{4}$

   (b) $1 + \imath$

   The modulus is $\sqrt{2}$, and the argument is 45^o, or $\frac{\pi}{4}$.

   (c) $e^{1+\frac{\pi}{3}\imath}$

   The modulus is e, and the argument is 60^o, or $\frac{\pi}{3}$.

3. Write the six 6^{th} roots of unity in $a + b\imath$ form.

$$\frac{1+\sqrt{3}}{2}$$

$$\frac{-1+\sqrt{3}}{2}$$

$$-1$$

$$\frac{-1-\sqrt{3}}{2}$$

$$\frac{1-\sqrt{3}}{2}$$

1

4. Write the five 5^{th} roots of unity in Polar form.

   $\frac{2\pi}{5}$ radians is 72^o.

   $\cos 72^o + \mathbf{i}\sin 72^o = \cos\frac{2\pi}{5} + \mathbf{i}\sin\frac{2\pi}{5}$

   $\cos 144^o + \mathbf{i}\sin 144^o = \cos\frac{4\pi}{5} + \mathbf{i}\sin\frac{4\pi}{5}$

   $\cos 216^o + \mathbf{i}\sin 216^o = \cos\frac{6\pi}{5} + \mathbf{i}\sin\frac{6\pi}{5}$

   $\cos 288^o + \mathbf{i}\sin 288^o = \cos\frac{8\pi}{5} + \mathbf{i}\sin\frac{8\pi}{5}$

   1

6. Write the eight 8^{th} roots of unity in $a + b\,\mathbf{i}$ form.

   $\frac{\sqrt{2}+\sqrt{2}\,\mathbf{i}}{2}$

   $\mathbf{i}$

   $\frac{-\sqrt{2}+\sqrt{2}\,\mathbf{i}}{2}$

   -1

   $\frac{-\sqrt{2}-\sqrt{2}\,\mathbf{i}}{2}$

   $-\mathbf{i}$

   $\frac{\sqrt{2}-\sqrt{2}\,\mathbf{i}}{2}$

   1

7. Write $e^{\frac{\pi}{8}\mathbf{i}}$ in $a + b\,\mathbf{i}$ form. Hint: you will need to compute complex square roots.

   This is the principle 16^{th} root of unity, with modulus 1 and argument $\frac{\pi}{8}$ radians, and is one of the square roots of $\frac{1+\mathbf{i}}{\sqrt{2}}$, which is more properly written $\frac{\sqrt{2}+\sqrt{2}\,\mathbf{i}}{2}$, since anything except a positive integer in the denominator is frowned upon. Using the formula for square roots of complex numbers, I got $\frac{\sqrt{2+\sqrt{2}}+\sqrt{2-\sqrt{2}}\,\mathbf{i}}{2}$.

   I verified this answer by squaring it.