University of Nevada, Las Vegas
Computer Science 477/677 Fall 2022
Final Examination December 15, 2022

Name:____

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

The entire test is 500 points.

1. Fill in the blanks.

   (i) [10 points] The following is pseudo-code for what algorithm?

   ____

```
int x[n];
obtain values of x;
 for(int i = n-1; i > 0; i++)
  for(int j = 0; j < i; j++)
   if(x[i] < x[j]) swap(x[i],x[j]);
```

   (ii) [10 points] ______________________ algorithm does not allow the weight of any arc to be negative.

   (iii) [10 points] The time complexity of every comparison-based sorting algorithm is ________________ . (Your answer should use Ω notation.)

   (iv) [10 points] The prefix expression $*a+\sim b*-c\,d\sim e$ is equivalent to the infix expression

   ____________________________ and the postfix expression ____________________________.

   (v) [10 points] The items stored in a priority queue represent ________________ ________________.

   (vi) [10 points] The asymptotic complexity of Dijkstra's algorithm algorithm is ______________________.

   (vii) [10 points] A ______________________ hash function fills the hash table exactly with no collisions.

   (viii) [10 points] In ________________ ________________ there can be any number of items at a given index of the hash table.

   (ix) [10 points] If the position at $h(x)$ is already occupied for some data item x, a ________________ ________________ is used to find an unoccupied position in the hash table.

   (x) [10 points] In ________________ hashing, each item has more than one hash value, but only uses one of them.

   (xi) [10 points] If G is a weighted directed graph, then it is impossible to solve the all pairs shortest path problem for G if G has a ______________________ ______________________

(xii) [10 points] A planar graph with n vertices can have no more than ________________ edges. (Exact formula, please.)

2. Give the asymptotic complexity, in terms of `n`, of each of the following code fragments. [10 points each]

(a)
```
for(int i = 2; i < n; i = i*i)
  cout << "Hello world" << endl;
```

(b)
```
for(int i = 1; i < n; i++)
  for(int j = 1; j < i; j = 2*j)
    cout << "Hello world" << endl;
```

(c)
```
for(int i = 1; i*i < n; i++)
  cout << "hello world" << endl;
```

(d)
```
for(int i = 0; i < n; i++)
  for(int j = n; j > i; j = j/2)
   cout << "Hello world!" << endl;
```

(e)
```
for(int i = 2; i < n; i=i*i)
  cout << "Hello world!" << endl;
```

(f)
```
for(int i = 1; i*i < n; i++)
  cout << "Hello world!" << endl;
```

(g)
```
for(int i = 1; i < n; i++)
  for(int j = 2; j < i; j=j*j)
    cout << "Hello world" << endl;
```

3. Solve the recurrences. Give the asymptotic value of $F(n)$ in terms of n, using Θ notation. [10 points each]

(a) $F(n) = 4F\left(\frac{n}{2}\right) + n$

(b) $F(n) \geq F\left(\frac{n}{2}\right) + 2F\left(\frac{n}{4}\right) + n$

(c) $F(n) \leq 2F(n/2) + n^2$

(d) $F(n) \geq 3F(n/9) + 1$

(e) $F(n) = F(3n/5) + 4F(2n/5) + n^2$

(f) $F(n) \leq 4F(n/2) + n^2$

(g) $F(n) \leq F(\sqrt{n}) + 1$

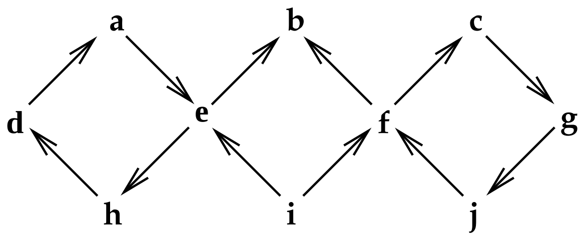
4. [20 points] List properties of a good hash function used for a search structure.

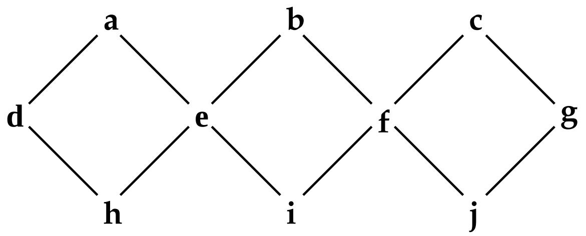
5. [20 points] Find an optimal prefix code for the alphabet $\{A, B, C, D, E, F\}$ where the frequencies are given in the following array.

A	4
B	5
C	2
D	6
E	8
F	1

6. [20 points] Write pseudocode for the Flyod/Warshall algorithm.

7. [20 points] Use the DFS method to find the strong components of the digraph in the first figure below. Show steps, using the second figure as needed.





8. [20 points] What is the loop invariant of the loop in the following function?

```
float product(float x, int n)
 {
  float z = 0.0;
  float y = x;
  int m = n;
  while(m > 0)
   {
    if(m%2) z = z+y;
    m = m/2;
    y = y+y;
   }
  return z;
 }
```

9. [20 points] Write pseudocode for the Bellman-Ford algorithm. Be sure to include the shortcut that ends the program when the final values have been found.

10. Consider the following C++ code.

```
int george(int n)
 {
  if(n == 0) return 1;
  else return george(n/2)+george(n/2-1)+n*n;
 }
```

(a) [10 points] What is the asymptotic complexity of `george(n)`?

(b) [10 points] What is the time complexity of the recursive code given above?

(c) [10 points] What is the time complexity of a dynamic programming algorithm to compute `george(n)`?

(d) [10 points] What is the space complexity of a computation of `george(n)` using memoization?

11. [20 points] Walk through mergesort with the array given below.

```
VJATNLDQMEFSPWGL
```

12. [20 points] Write pseudcode for the simple coin-row problem we discussed in class. Your are given a row of n coins of various values. The problem is to select a set of coins of maximum total value, subject to the condition that no two adjacent coins are selected. Your code should identify the coins which are selected.

13. [20 points] Write pseudocode for a function `float power(float x, int n)` that returns x^n. You may assume that $x! = 0$, but your code must work for any integer n. It is not necessary to use the algorithm given in class; use any algorithm that works.

14. [20 points] The following code is used as a subroutine for both quicksort and select. Assume A[n] is an array of integers. For simplicity, we assume that no two entries of A are equal. Write a loop invariant for the while loop.

```
int pivot = A[0];
int lo = 0;
int hi = n-1;
while(lo < hi)
 {
  if(A[lo+1] < pivot) lo++;
  else if(A[hi] > pivot) hi--;
  else swap(A[lo+1],A[hi]);
 }
```