University of Nevada, Las Vegas Computer Science 477/677 Spring 2024

Answers to Assignment 1: Due Friday September 6, 2024

1. Problem 0.1 on page 8 of Dasgupta, Papadimitriou and Vazirani. Write either O, Ω or Θ in each blank. Do not write O or Ω if Θ is correct.

   (a) $n - 100 = \Theta\,(n - 200)$

   (b) $n^{1/2} = O\,(n^{2/3})$

   (c) $100n + \log n = \Theta\,(n + \log^2 n)$

   (d) $n \log n = \Omega\,(10n + \log(10n))$

   (e) $\log(2n) = \Theta\,(\log(3n))$

   (f) $10 \log n = \Theta\,(\log(n^2))$

   (g) $n^{1.01} = \Omega\,(n \log^2 n)$

   (h) $n^2 / \log n = \Omega\,(n \log^2 n)$

   (i) $n^{0.1} = \Omega\,(\log^2 n)$

   (j) $(\log n)^{\log n} = \Omega\,(n / \log n)$

   (k) $\sqrt{n} = \Omega\,(\log^3 n)$

   (l) $n^{1/2} = O\,(5^{\log_2 n})$

   (m) $n2^n = O\,(3^n)$

   (n) $2^n = \Theta\,(2^{n+1})$

   (o) $n! = \Omega\,(2^n)$

   (p) $\log_2 n^{\log_2 n} = O\,(2^{(\log_2 n)^2})$

   (q) $\sum_{i=1}^{n} i^k = \Theta\,(n^{k+1})$

2. The following problem is a modified version of problem 0.3(c) on page 9 of DasGupta, Papadimitriou, and Vazirani. The answer is not exactly the same as the answer to the textbook version, but the techniques are similar.

$F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. Find the smallest constant C such that $F_n = O(C^n)$. We start by assuming $F_n = n^C$ for some constant C. This is false, but it's true in the limit; that is, for larger and larger values of n, the ratio of F_n to C^n converges to 1. That is, there is a constant C such that $\lim_{n\to\infty} \frac{F_n}{C^n} = 1$. Making the assumption that they are actually equal:

$$\begin{aligned} F_{n+2} &= F_{n+1} + F_n \\ C^{n+2} &= C^{n+1} + C^n \\ \text{Divide both sides by } C^n: & \\ C^2 &= C^1 + C^0 \end{aligned}$$

Which is a quadratic equation over C. The quadratic formula gives us two solutions: $C = \frac{1\pm\sqrt{5}}{2}$ But C is not the negative solution, since $\{F_n\}$ would converge to zero. Thus $C = \frac{1+\sqrt{5}}{2}$ which is the so-called *golden ratio.* Since we did make an incorrect assumption, we might worry about this answer. But it's correct. Classic Greek mathematicians knew about the Golden Ratio. The base of the Parthenon in Athens is a rectangle whose longer dimension is $\frac{1+\sqrt{5}}{2}$ times the smaller dimension.

3. Consider the following C++ program.

```
void process(int n)
 {
  cout << n << endl;
  if(n > 1) process(n/2);
  cout << n%2;
 }

int main()
 {
  int n;
  cout << "Enter a positive integer: ";
  cin >> n;
  assert(n > 0);
  process(n);
  cout << endl;
  return 1;
 }
```

The last line of the output of `process(n)` is a string of bits. What does this bitstring represent?

The binary numeral for n.

4. The C++ code below implements a function, "mystery." What does it compute?

```
float squre(float x)
 {
  return x*x;
 }

float mystery(float x, int k)
 {
  if (k == 0) return 1.0;
  else if(x == 0.0) return 0.0;
  else if (k < 0) return 1/mystery(x,-k);
  else if (k%2) return x*mystery(x,k-1);
  else return mystery(squre(x),k/2);
 }
```

It computes x^k.