Recurrences: Akra Brazzi Method

We consider recurrences of the following form:

$F(n) = \sum_{i=1}^{k} a_i F(b_i n) + n^c$

where

- $a_i > 0$ for each i
- $0 < b_i < 1$ for each i
- $c \geq 0$

First, compute $\sum_{i=1}^{k} a_i b_i^c$. There are three formulae for the solution.

Case 1: $\sum_{i=1}^{k} a_i b_i^c < 1$

$F(n) = \Theta(n^c)$

Case 2: $\sum_{i=1}^{k} a_i b_i^c = 1$

$F(n) = \Theta(n^c \log n)$

Case 3: $\sum_{i=1}^{k} a_i b_i^c > 1$

First, find an exponent d such that $\sum_{i=1}^{k} a_i b_i^d = 1$. Then

$F(n) = \Theta(n^d)$.

Examples

(i) $F(n) = F(n/2) + F(n/3) + F(n/6) + n^2$

(ii) $F(n) = F(n/2) + F(n/3) + F(n/6) + n$

(iii) $F(n) = F(n/2) + F(n/3) + F(n/6) + 1$

(iv) $F(n) = F(n/9) + F(4n/9) + \sqrt{n}$

(v) $F(n) = 3F(n/3) + 3F(2n/3) + n^2$