University of Nevada, Las Vegas Computer Science 477/677 Fall 2023

Answers to Examination September 27, 2023

1. Simplify each expression.

   (a) $\log_9 3 = \frac{1}{2}$ since $9^{\frac{1}{2}} = 3$

   (b) $2^{\log 5} = 5$ because in this course log defaults to $\log_2$.

2. Fill in the blanks.

   (a) Any comparison-based sorting algorithm on a file of size n must execute $\Omega(\log n!) = \Omega(n \log n)$ comparisons in the worst case.

   (b) Name two divide-and-conquer sorting algorithms.

   quicksort

   mergesort

3. The items in a priority queue, such as a stack, represent unfulfilled obligations. In class we discussed a stack algorithm for converting an infix expression to an equivalent postfix expression.

   Each operator on the stack represents the unfulfilled obligation to **write that operator**

   Each left parenthesis on the stack represents the unfulfilled obligation to **read a right parenthesis**

4. You have an array consisting of thousands of names in alphabetical order. What algorithm would you use to determine whether this array contains the name "Arthur Linkletter"?

   binary search

5. Find the time complexity of each of these code fragments in terms of n, using Θ notation.

   (a)
   ```
   for(int = 0; i < n; i++)
    for(int j = i; j > 0; j=j/2);
   ```

   The inner loop executes $\Theta(\log i)$ times. Replacing i by the real variable x we can approximate the time complexity by the integral

   $$\int_1^n \ln x dx = [x \ln x - x]_1^n = n \ln n - n + 1 = \Theta(n \log n)$$

   (b)
   ```
   for(int i = 0; i < n; i++)
    for(int j = n; j > i; j=j/2);
   ```

   For each iteration of the outer loop, the inner loop executes approximately $\log n - \log i$ times. Substituting the real variable x for i, we approximate the time complexity by the integral

   $$\int_1^n (\ln n - \ln x)dx = [xlnn - xlnx + x]_1^n = n \ln n - n \ln n + n - \ln n - 1 = \Theta(n)$$

(c)
```
for(int i = 1; i < n*n; i = 2*i)
 for(int j = i; j > 0; j=j-1);
```

Let $k = \log i$ and $m = \log n$. Thus $2^k = i$ and $2^m = n$; hence $2^{2m} = n^2$. Substituting, we have

```
for(int k = 0; k < 2m; k++)
 for(int j = 2^k; j > 0; j=j-1);
```

Recall (from calculus) that $\int 2^x dx = \dfrac{2^x}{\ln 2} + C$

The inner loop executes $\Theta(2^k)$ times. Replacing k by the real variable x, the time complexity is approximated by the integral

$$\int_0^{2m} 2^x dx = \left[\frac{2^x}{\ln 2}\right]_0^{2m} = \frac{2^{2m}-1}{\ln 2} = \Theta(2^{2m} = \Theta(n^2)$$

(d)
```
for(int i = 1; i < n; i++)
 for(int j = 1; j < i*i; j++);
```

The inner loop executes i^2 times. Replacing i by x, we have

$$\int_1^n x^2 dx = \left[\frac{x^3}{3}\right]_1^n = \frac{n^3-1}{3} = \Theta(n^3)$$

(e)
```
for(int i = 1; i < n; i=2*i)
 for(int j = 2; j < i; j=j*j)
```

Let $k = \log i$, $m = \log n$, and $l = \log j$. We have

```
for(int k = 0; k < m; k++)
 for(int l = 1; l < k; l=2*l)
```

The inner loop executes $\Theta(\log k)$ times. Replacing k by x we approximate the time complexity by

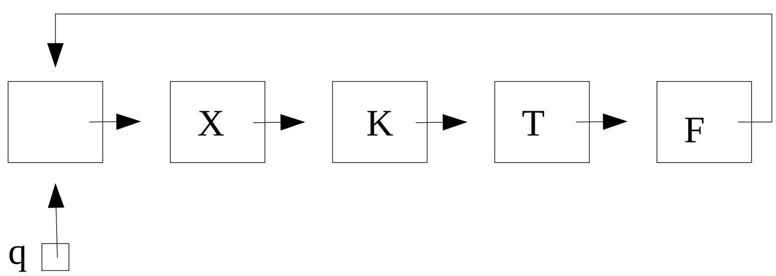
$$\int_1^m \ln x dx = [x \ln x - x]_1^m = \Theta(m \log m) = \Theta(\log n \log\log n)$$

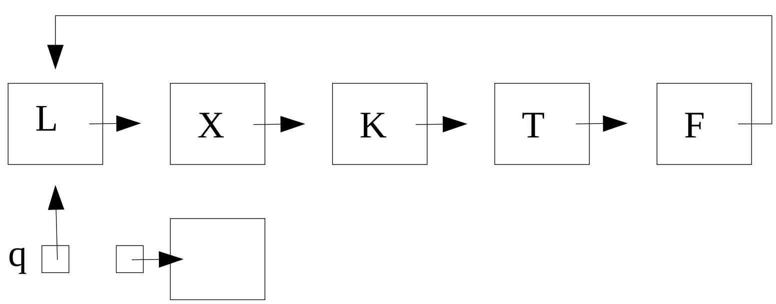
6. Find the loop invariant of the following C++ function, which computes $\lfloor \log n \rfloor$.

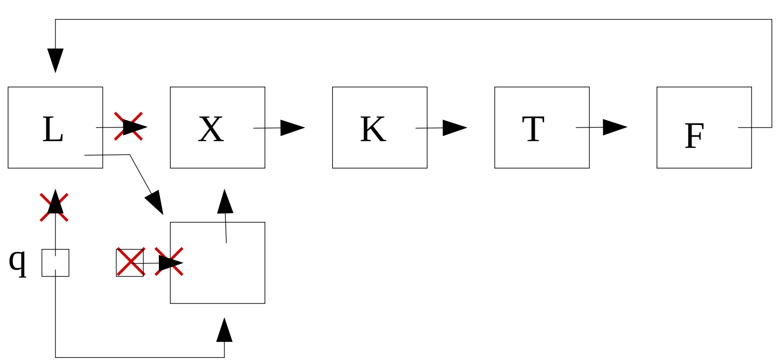
```
int flooroflogarithm(int n)
 {
  // input condition: n > 0
  int m = n;
  int rslt = 0;
  while(m > 1)
   {
    rslt++;
    m = m/2;
   }
  return rslt;
 }
```

The procedure approximates $\log n$ by repeatedly halving n until it reaches 1. The variable rslt counts the number of times it is halved. At any intermediate point, the value of m is the orginal n halved rslt times. Thus the loop invariant is $\lfloor \log n \rfloor = \lfloor \log m \rfloor +$ rslt

7. Draw a circular queue with dummy node, holding items X, K, T, F, in that order from front to rear. Draw figures illustrating how the queue changes when you insert L.







8. A stack of integers is implemented in C++ as a linked list follows.

```
struct stacknode
 {
  int item;
  stacknode*link;
 };
 typedef stacknode*stack;
```

Write code for the operators push, pop, and empty.

```
void push(stack&s,int newitem)
 {
  stack temp = new stacknode;
  temp->item = newitem;
  temp->link = s;
  s = temp;
 }


bool empty(stack s)
 {
  return s == NULL;
 }


int pop(stack&s)
 {
  assert(not empty(s));
  int rslt = s->item;
  s = s->link;
  return rslt;
 }
```

9. Walk through the steps of heapsort, sorting an array whose items are R,U,W,F,B,Y in that order. For ease of grading, use the matrix below. (You don't need all the rows.)

1	2	3	4	5	6	
R	U	W	F	B	Y	initial array
R	U	Y	F	B	W	bubbledown W
Y	U	R	F	B	W	bubbledown R
Y	U	W	F	B	R	bubbledown R, heapify completed
R	U	W	F	B	**Y**	swap first and last
W	U	R	F	B	**Y**	bubbledown R, heap order restored
B	U	R	F	**W**	**Y**	swap first and last
U	B	R	F	**W**	**Y**	bubbledown B
U	F	R	B	**W**	**Y**	bubbledown B, heap order restored
B	F	R	**U**	**W**	**Y**	swap first and last
R	F	B	**U**	**W**	**Y**	bubbledown B, heap order restored
B	F	**R**	**U**	**W**	**Y**	swap first and last
F	B	**R**	**U**	**W**	**Y**	bubbledown B, heap order restored
B	**F**	**R**	**U**	**W**	**Y**	swap first and last
B	**F**	**R**	**U**	**W**	**Y**	sorted