Computer Science 477/677 Fall 2002 Examination, September 26, 2002

Name:_____

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

The entire test is 125 points.

1. Fill in the blanks. [5 points each blank.]

   (a) A directed graph is called a tree if there is exactly one _____ from any given vertex to a special vertex called the _____.

   (b) A graph is called _____ if, given any two vertices, there is a path between them.

   (c) Suppose a heap is implemented as an array, where the root is stored in position 1. What are the two array positions for the children of the item stored in position 6? _____

   (d) A path is called _____ if it uses no vertex more than once.

2. A stack is implemented using an array `A` and a variable `n` which indicates the number of items currently in the stack. The items are stored in positions `A[0] ... A[n-1]`. Write code which pushes a new item, `x`, onto the stack. Do not check for overflow. You should have only **two** lines of code. [20 points]

3. Solve the recurrence $f(n) \geq f(n/2) + 2n^2$. Give an asymptotic answer in terms of n, using either O, Ω, or Θ, whichever is most appropriate. [10 points]

4. Consider the following procedure:

```
void george(int n)
 {
   if (n > 1)
    {
      for (int i = 1; i < n; i++)
       cout << "hello world" << endl;
      george(n/2);
      george(n/2);
    }
 }
```

How many lines of output would execution of `george(n)` produce? Write down an appropriate recurrence for this question, and give an asymptotic solution in terms of `n`, using either O, Ω, or Θ, whichever is most appropriate. [20 points]

5. What are the important characteristics of a good hash function? [15 points]

6. Give a **mathematically correct** definition of the statement, "$f(n) = O(n^2)$." (If you write more than 15 words, your answer is probably wrong. I will take off points if you give an example, or write anything else that is unnecessary.) [15 points]

7. Consider the following array representing a weighted directed graph G.

$$\begin{bmatrix} \infty & 2 & \infty & \infty \\ 5 & \infty & 3 & \infty \\ \infty & \infty & 1 & 0 \\ 1 & \infty & \infty & \infty \end{bmatrix}$$

(a) Draw a picture of G. [5 points]

(b) Use matrix operations to compute the reflexive transitive closure G^*, showing your work. [15 points]