Computer Science 477/677 Fall 1998 Midterm Exam, October 3, 1998

Name:_____

No books, notes, or scratch paper. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

The entire test is 90 points.

1. True or False. [5 points each]

   (a) A tree *must* have at least one node. _____

   (b) According to our textbook, a node in a rooted tree is an ancestor of itself. _____

   (c) It is possible for the root of a tree to also be a leaf. _____

   (d) A binary tree *must* have at least one node. _____

2. Solve the following recurrences. [10 points each] In each case, give the answer in terms of Θ if possible. Otherwise, give the answer in terms of either O or Ω, whichever is appropriate.

   (a) $f(n) = 2f(\sqrt{n}) + \log n$

   (b) $g(n) = 2g(n-1) + 1$

3. Fill in each black space with exactly one word or formula. [5 points each]

   (a) A maximal connected subgraph of a graph G is called a _____ of G.

   (b) A graph which has no cycle is called _____.

   (c) In a tree, a node which has no children is called a _____.

   (d) If a graph has n nodes and _____ edges it is called a *complete* graph.

   (e) A directed graph is called _____ connected if there is a directed _____ from any node to any other node.

   (f) If G is a graph and S is a _____ of G which is a tree and which contains every node of G,

       then we say that S is a _____ tree of G.

4. The following code is for a recursive procedure. You many assume that the actual parameter is a non-negative integer. Let $T(n)$ be the execution time for the procedure given parameter n. Write a recurrence for T, and solve it, expressing the answer with either Θ, O, or Ω as appropriate. [20 points] You may assume that n/2 is integer division, as in C or C++.

```
proc george(n)
   if (n > 0) then
      george(n/2)
```

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5. Consider the algorithm Heapsort. Suppose that initially an array A has the values $A = \langle 3, 4, 8, 1, 0, 9, 2, 7, 6, 5 \rangle$ What will A be after executing BUILD-HEAP(A)? [20 points]

6. Let T be a binary tree. Let ℓ be the number of leaves of T, and let b be the number of nodes of T which have exactly two children. Prove by induction that $b + 1 = \ell$. [30 points] (Hint: First draw some binary trees to convince yourself that the statement is true. Then use the recursive definition of binary tree and the Strong Inductive Principle.)