

Computer Science 477/677 Spring 2000 Examination, February 12, 2000

Name: \_\_\_\_\_

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

The entire test is 100 points.

1. Fill in the blanks. [5 points each blank.]

- (a) \_\_\_\_\_ and \_\_\_\_\_ are **divide and conquer** sorting algorithms.
- (b) The time complexity of any algorithm to sort  $n$  items is \_\_\_\_\_ under the **decision tree** model of computation
- (c) What is  $\lfloor \log_2 5000 \rfloor$ ? \_\_\_\_\_ Give an exact answer. Remember: do **not** use a calculator.
- (d) The worst case asymptotic time complexity of **heapsort** is \_\_\_\_\_.
- (e) The worst case asymptotic time complexity of **quicksort** is \_\_\_\_\_.

2. Give a **mathematically correct** definition of the statement, " $f(n) = \Omega(g(n))$ ," under the assumption that the **domain** of  $g$  is the **natural numbers**, that is, the non-negative integers, that  $g(n) \geq 0$  for every natural number  $n$ , and that the domain of  $f$  is also the natural numbers. [15 points]

3. Solve the following recurrences. [10 points each]

(a) Suppose that  $F(n) = 2 \cdot F\left(\frac{n}{2}\right) + 5n + 100$ . Write the **asymptotic complexity** of  $F(n)$  using either  $O$  notation,  $\Omega$  notation, or  $\Theta$  notation, whichever is most appropriate for this problem.

(b) Suppose that  $F(n) \leq 1 + 2 \cdot F(n - 1)$ . Write the **asymptotic complexity** of  $F(n)$  using either  $O$  notation,  $\Omega$  notation, or  $\Theta$  notation, whichever is most appropriate for this problem.

4. We are given an array  $A[i]$ , where the indices range from 1 to the some constant which is as least as large as the value of some variable  $n$ . The entries of  $A$  have type `real`. What are the conditions on  $A$  that must be satisfied, so that  $A$  is the array for an array implementation of a **heap** of size  $n$ ? [15 points]

5. Consider the following pseudocode subprogram. Determine the asymptotic time complexity (expressed in terms of  $n$ , the parameter) of the computation  $f(n)$ . You may assume that the value of the parameter is a natural number.

You must set up the appropriate recurrence and then give the solution. Write the asymptotic time complexity using either  $O$  notation,  $\Omega$  notation, or  $\Theta$  notation, whichever is most appropriate for this problem. [20 points]

```
f(integer n)
{
  for all integers i in the range 1 to 4*n, write(i);
  integer m = n/2; // comment: use truncated division, e.g., 7/2 = 3
  if (m > 0) then f(m);
}
```