Computer Science 477/677 Spring 1999 Final Exam 6:00 PM, Monday, May 10, 1999, TBE B174

Name:_____

No books, notes, or scratch paper. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

The entire test is 270 points.

1. True or false. [5 points each.]

   (a) _____ Computers are so fast nowadays that discussion of time complexity is obsolete.

   (b) _____ One of the operators of a Heap is *find*.

2. Fill in the blanks. [5 points each blank.]

   (a) _____ is known to be an $\mathcal{NP}$-complete problem.

   (b) _____ programming is a technique whereby a family of subproblems is solved in an appropriate order, such that the solution of each subproblem may depend on the solutions of prior subproblems.

   (c) A _____ of a graph G is a maximal non-empty connected subgraph of G.

   (d) _____ [three words] and _____ [three words] are two standard technique of visiting the nodes of a directed graph.

   (e) _____ order is any order of an acyclic directed graph such that if e is an edge from node x to node y, then node x comes before y in the order.

   (f) _____ is an order of the nodes of an ordered tree such that, if node x is the parent of nodes y and z, and if y is to the left of z, then y comes before z and z comes before x in the ordering.

   (g) A binary tree of height h must have at most _____ nodes. [**Exact** formula, please.]

3. Solve each of the following recurrences [10 points each], giving each answer in Θ notation.

(a) $f(n) = 2f(\frac{n}{2}) + 3n$

(b) $g(n) = g(\sqrt{n}) + 1$

(c) $T(n) = 2T(n-1) + 1$

(d) $h(n) = n + h(\frac{1}{3}n) + h(\frac{3}{5}n)$

4. What is a *spanning tree* of a graph? [10 points]

5. Give the three standard operators for the ADT *stack*, other than a constructor operator. Just the names of the operators, please. [10 points]

6. Give the two standard operators for the ADT *array*, other than a constructor operator. Just the names of the operators, please. [10 points]

7. Give the three standard operators for the ADT *search structure*, other than a constructor operator. Just the names of the operators, please. [10 points]

8. Give four different implementations of the ADT *search structure*, all very different from one another. (Do not describe them in detail, unless they are not ones that we discussed in class. This should not be necessary, since we discussed at least four in class.) [20 points]

9. Sketch a sorting algorithm which sorts an array A of length n in $O(n^2)$ time but uses only constant space other than the space used to store A itself. [15 points]

10. Sketch Mergesort. [15 points]

11. Sketch Quicksort. [30 points]

12. Sketch an algorithm which does *not* use division, and which determines whether two line segments in a plane intersect. The input of the algorithm is four points: p_1, p_2, p_3, and p_4. The output is a single bit, which is 1 if the line segment $\overline{p_1 p_2}$ intersects the line segment $\overline{p_3 p_4}$, and zero otherwise. [30 points]

13. Sketch Dijkstra's algorithm for solving the single source shortest path problem for a weighted directed graph with no negative weights. [30 points]