Computer Science 477/677 Spring 1999 Examination, February 17, 1999

Name:_____

No books, notes, or scratch paper. Use pen or pencil, any color. Use the rest of this page and the backs of the pages for scratch paper. If you need more scratch paper, it will be provided.

The entire test is 140 points.

1. Fill in the blanks. [5 points each blank.]

   (a) _____ and _____ are "divide and conquer" sorting techniques.

   (b) True or false: Computers are so fast nowadays that it doesn't matter whether programs are efficient. _____

   (c) If you enter a positive number n on your calculator, and then divide by 2 repeatedly until the number is less than or equal to 1, what is the *exact* number of times you will divide by 2? _____

2. Give a mathematically correct definition of the statement, "$f(n) = O(n^2)$" [10 points]

3. Explain, in English, how it is that Heapsort is a version of Selection Sort. [10 points]

4. Solve the following recurrences. [10 points each]

(a) Suppose that $T(n) = T\left(\frac{n}{2}\right) + \sqrt{n}$. Write the asymptotic complexity of $T(n)$ using Θ notation.

(b) Suppose that $F(n) \leq 1 + F(\sqrt{n})$. Write the asymptotic complexity of $T(n)$ using O notation.

(c) Suppose that $G(n) \geq 5n + G\left(\frac{n}{2}\right) + G\left(\frac{n}{3}\right)$. Write the asymptotic complexity of $G(n)$ using Ω notation.

5. Explain how the array implementation of the ADT "stack" works. You may use pseudocode to explain what you are doing. [20 points]

6. What are the operators of the ADT "priority queue"? Explain each operator. [20 points]

7. We discussed *Loop Invariants* in class.

   (a) What is a loop invariant? [10 points]

   (b) Consider the SPLIT procedure used in Quicksort. Write pseudo-code for that procedure. [10 points]

   (c) Identify the loop invariant in the pseudocode for the version of SPLIT that you wrote above. (You may use a picture to illustrate the loop invariant, but you must state it in words as well.) [10 points]