Name:______________________________________________________________

No books, notes, scratch paper, or calculators. Use pen or pencil, any color. Use
the rest of this page and the backs of the pages for scratch paper. If you need
more scratch paper, it will be provided.

The entire test is 135 points.

1. True or False. [5 points each]

   (a) ______ Computers are so fast today that complexity theory is only of theoretical, but not practical,
       interest.

   (b) ______ If any problem can be precisely formulated in a mathematical way, there is an algorithm
       that solves it.

   (c) ______ Heapsort takes $\Theta(n \log n)$ time on an array of size $n$.

2. Fill in the blanks. [5 points each blank.]

   (a) What is the only difference between the abstract data types queue and stack?

       _______________________________________________________________

   (b) Name a divide-and-conquer searching algorithm.

       _______________________________________________________________

   (c) Name two divide-and-conquer sorting algorithms.

       _______________________________________________________________
       _______________________________________________________________

   (d) The following is pseudo-code for which sorting algorithm we’ve discussed?

       _______________________________________________________________

       int x[n];
       for(int i = n-1; i > 0; i--)
       
           Find the largest element of x[0], ... x[i] and swap it with x[i]
3. Write the following asymptotic complexity classes in order, using “=” to mean that two classes are exactly the same, and “⊂” to mean that one class is a proper subset of the other; log means log₂.

\[ O(2^{\log n}), O(4n + 3), O(2^n), O(F_n) \text{ (the } n^{\text{th}} \text{ Fibonacci number}), O(n \log n), O(\log^2 n), O(\log n), O(n^{1.1}), O(1.1^n) \quad O(n^3), O(\log 2), O\left(\frac{1}{n}\right) \]

4. Solve each of the following recurrences, giving the answer in terms of \( O, \Theta, \) or \( \Omega, \) whichever is most appropriate [10 points each].

(a) \( T(n) < T(n - 2) + n^2 \)

(b) \( F(n) \geq F(\sqrt{n}) + \log n \)

(c) \( G(n) \geq G(n - 1) + n \)
(d) $F(n) = 4F(n/2) + n^2$.

(e) $H(n) \leq 2H(\sqrt{n}) + O(\log n)$.

(f) $K(n) = K(n - \sqrt{n}) + 1$.

(g) $F(n) = 4F(\frac{3n}{4}) + n^5$ \hspace{1cm} (No, you don't need a calculator.)
5. [15 points] Consider the following procedure:

```cpp
void george(int n)
{
    int m = n;
    while (m > 1)
    {
        for (int i = 1; i < m; i++)
            cout << "I cannot tell a lie. I chopped down the cherry tree.\n" << endl;
        m = m/2;
    }
}
```

Consider the question of how many lines of output the execution of `george(n)` would produce. Write down an appropriate recurrence for this question, and give an asymptotic solution in terms of `n`, using either $O$, $\Omega$, or $\Theta$, whichever is most appropriate.

6. Write correct pseudocode (or C++ code) for __________, which could be:
   (a) Bubblesort an array of size $n$.
   (b) Selectionsort an array of size $n$.
   (c) The partition loop of quicksort.