University of Nevada, Las Vegas Computer Science 477/677 Spring 2020

Answers to Assignment 2: Due Tuesday February 11, 2020

1. Work problem 2.5 on page 71 of the texctbook. Do not replace any transcendental constant with a decimal. For example "$\log_2 3$" should be left as is, but "$\log_2 4$" should be written as 2.

   (a) $T(n) = 2T(n/3) + 1$
   $T(n) = \Theta\left(n^{\log_3 2}\right)$

   (b) $T(n) = 5T(n/4) + n$
   $T(n) = \Theta\left(n^{\log_4 5}\right)$

   (c) $T(n) = 7T(n/7) + n$
   $T(n) = \Theta(n \log n)$

   (d) $T(n) = 9T(n/3) + n^2$
   $T(n) = \Theta(n^2 \log n)$

   (e) $T(n) = 8T(n/2) + n^3$
   $T(n) = \Theta(n^3 \log n)$

   (f) $T(n) = 49T(n/25) + n^{3/2} \log n$
   $T(n) = \Theta(n^{3/2} \log n)$

   (g) $T(n) = T(n-1) + 2$
   $T(n) = \Theta(n)$

   (h) $T(n) = T(n-1) + n^c$ where $c \geq 1$ is a constant.
   $T(n) = \Theta(n^{c+1})$

   (i) $T(n) = T(n-1) + c^n$ where $c > 1$ is a constant.
   $T(n) = \Theta(c^n)$

   (j) $T(n) = 2T(n-1) + 1$ Work this one by substitution. Let $F(m) = T(\log_2 m) = T(n)$. Then $T(n-1) - T(\log_2 m - 1) = T(\log_2(m/2)) = F(m/2)$

   Substituting, we have $F(m) = 2F(m/2) + 1$. Thus

   $$T(n) = F(m) = \Theta(m) = \Theta(2^n)$$

   (k) $T(n) = T(\sqrt{n}) + 1$ Use substitution: $m = \log_2 n$,

   Substituting $T(n) = F(m) = F(\log n)$ and $F(m) = F(m/2) + 1$ Thus

   $$T(n) = F(m) = \Theta(\log m) = \Theta(\log \log n)$$

2. Work problem 2.4 on page 71 of the textbook.

   Algorithm C takes $\Theta(n^2 \log n)$ time, which is faster than the others.

3. Problem 2.12 on page 73 of the textbook.

   $T(n) = 2T(n/2) + 1$.

   $T(n) = \Theta(n)$