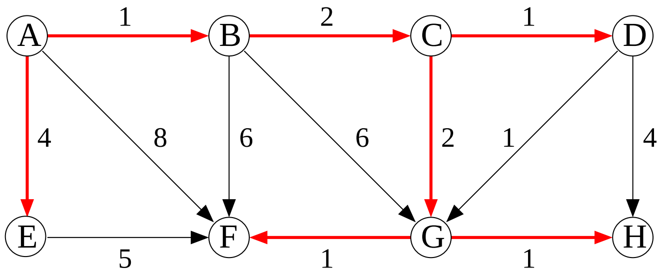
University of Nevada, Las Vegas Las Vegas Computer Science 477/677 Spring 2020

Answers to Assignment 5: Due Tuesday March 24, 2020

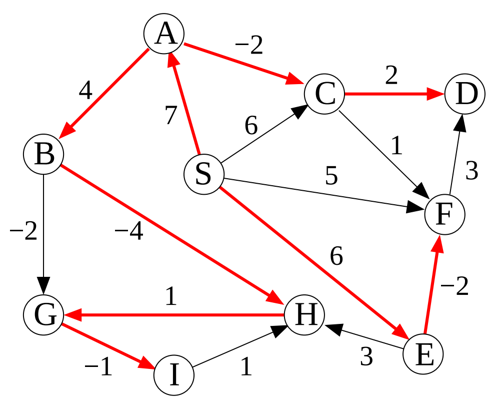
1. Work Problem 4.1 on page 120 of your textbook.



	0	1	2	3	4	5	6	7	8
A	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
B	∞	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
C	∞	∞	**3**	**3**	**3**	**3**	**3**	**3**	**3**
D	∞	∞	∞	**4**	**4**	**4**	**4**	**4**	**4**
E	∞	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**
F	∞	**8**	**7**	**7**	**7**	**7**	**6**	**6**	**6**
G	∞	∞	**7**	**5**	**5**	**5**	**5**	**5**	**5**
H	∞	∞	∞	∞	∞	**8**	**6**	**6**	**6**

Each entry shows the current best distance from A of that node at that step. The colors of the table encode the data structure. A red numeral indicates that the corresonding node is in the priority queue at that step. numbers may be changed. A black or blue numeral indiates the minimum distance from A to that node. A blue numeral indicates that the corresponding node is deleted from the priority queue at that step, and its subsequents are visited and their The shortest path tree is shown in red.

2. Work Problem 4.2 on page 120 of your textbook.



	0	1	2	3	4	5	6
S	0	0	0	0	0	0	0
A	∞	7	7	7	7	7	7
B	∞	∞	11	11	11	11	11
C	∞	6	5	5	5	5	5
D	∞	∞	8	7	7	7	7
E	∞	6	6	6	6	6	6
F	∞	5	4	4	4	4	4
G	∞	∞	∞	9	8	8	8
H	∞	∞	9	7	7	7	7
I	∞	∞	∞	∞	8	7	7

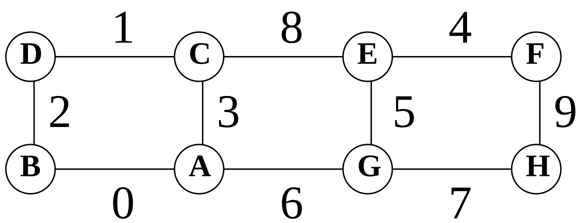
The shortest path tree is shown in red.
There are two choices for the shortest path tree.

3. You are working on computer which lacks multipliation and addition. However, it can add or subtract 1 or 2. What does this function do? What is its loop invariant?

```
int double(int n)
 // input condition: n >= 0
 {
  int p = n;
  int q = 0;
  while(p > 0)
   {
    p = p-1;
    q = q+2;
   }
  return q;
 }
```

The function returns $2n$. The loop invariant is: $p \geq 0$ and $2p + q = 2n$.

4. Walk through Kruskal's algorithm on the graph shown below. Show the union/find data structure after each step.



We execute a step for each edge, in order of weight. The step for the edge {X,Y} is

```
void Step{vertex X,vertex Y)
 {
  vertex U = find(X);
  vertex U = find(Y);
  if(U != V)
   union(U,V);
 }
```

Each set is represented as a tree rooted at its leader, Initially, each set has rank 0 and consists of a single vertex. When we execute union(U,V) where U and V are distinct leaders, we combine their trees by making V the parent of U if rank(U) ¡ rank(V), or if they have equal rank and V is after U in the order of vertices, alphabetic for this case. If V retains the same rank if rank(U) ¡ rank(V); if they have equal rank, rank(V) is incremented by 1.

(a) In the initial configuration, each vertex is a leader and has rank 0.

(b) Execute Step(A,B). B is now the parent of A. A acquires rank 1. In the figures, I delete rank labels for all vertices except leaders, to indicate that those values are now irrelevant.

(c) Execute Step(C,D). D becomes the parent of C, and gets rank 1.

(d) Execute Step (B,D). D becomes the parent of B, and gets rank 2.

(e) Execute Step (A,C). Since find(A) = find(C), there is no union. However, find(A) triggers path compression, and the parent of A becomes D.

(f) Execute Step(E,F). The parent of E becomes F.

(g) Execute Step(E,G). The parent of G becomes F, since find(E) = F.

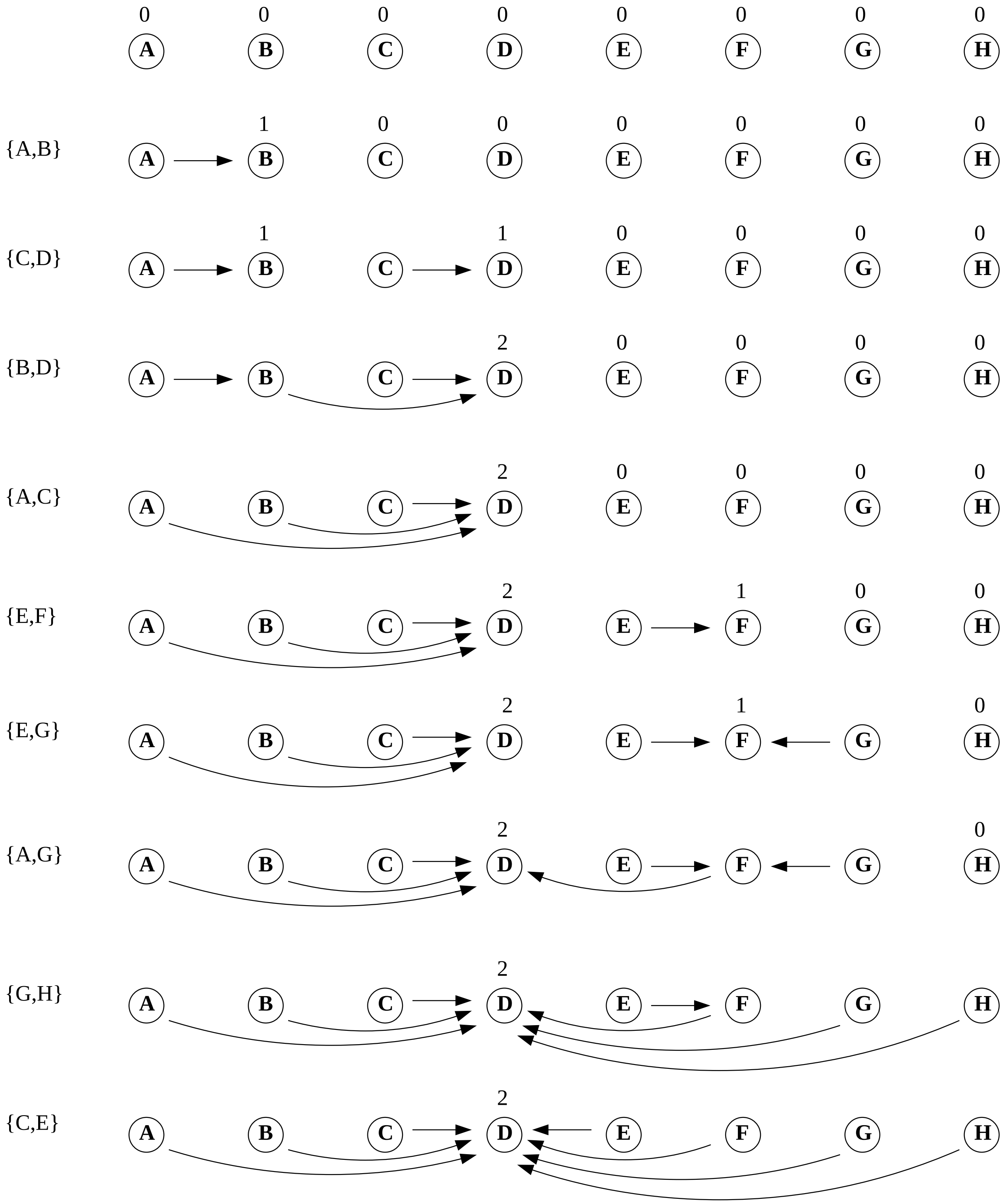
(h) Execute Step(A,G). Find(A) = D and find(G) = F. Parent(F) becomes D.

(i) Execute Step(G,H). Find(G) = D, causing parent(G) = D by path compression, and parent(H) becomes D. There is no union. All vertices now belong to the same set, but we may continue.

(j) Execute Step(C,E). There is no union, but find(E) triggers path compression.

(k) Step(F,H) does not change the data structure.

(l) An edge {X,Y} becomes part of the spanning tree if there is a union during the execution of Step(X,Y). The resulting spanning tree consists of the emboldened edges in the last figure..


{A,B}
{C,D}
{B,D}
{A,C}
{E,F}
{E,G}
{A,G}
{G,H}
{C,E}