University of Nevada, Las Vegas Computer Science 477/677 Spring 2020

Assignment 1: Due Tuesday January 28, 2020

Name:_____

You are permitted to work in groups, get help from others, read books, and use the internet. But the handwriting on this document must be your own. Print out the document, staple, and fill in the answers. You may attach extra sheets. Turn in the pages to the graduate assistant at the beginning of class, August 14.

1. Problem 0.1 on page 8 of the textbook. In each of the following situations, write O, Ω. Θ in the blank.

   (a) $n - 100 =$ _____ $(n - 200)$

   (b) $n^{1/2} =$ _____ $(n^{2/3})$

   (c) $100n + \log n =$ _____ $(n + \log^2 n)$

   (d) $n \log n =$ _____ $(10n \log(10n))$

   (e) $\log(2n) =$ _____ $(\log(3n))$

   (f) $10 \log n =$ _____ $(\log(n^2))$

   (g) $n^{1.01} =$ _____ $(n \log^2 n)$

   (h) $n^2/\log n =$ _____ $(n \log^2 n)$

   (i) $n^{0.1} =$ _____ $(\log^2 n)$

   (j) $(logn)^{\log n} =$ _____ $(n/\log n)$

   (k) $\sqrt{n} =$ _____ $(\log^3 n)$

   (l) $n^{1/2} =$ _____ $(5^{log_2 n})$

   (m) $n2^n =$ _____ (3^n)

   (n) $2^n =$ _____ (2^{n+1})

   (o) $n! =$ _____ (2^n)

   (p) $\log n^{\log n} =$ _____ $(2^{(\log_2 n)^2})$

   (q) $\sum_{i=1}^{n} i^k =$ _____ (n^{k+1})

2. Work problem 0.3(c) on page 9 of the textbook.

3. Consider the following C++ program.

```
void process(int n)
 {
  cout << n << " " << n%2 << endl;
  if(n > 1) process(n/2);
 }

int main{)
 {
  int n;
  cin >> n;
  process(n);
 }
```

For example, if you enter the value 37, the output will look like this:

```
37 1
18 0
 9 1
 4 0
 2 0
 1 1
```

For any positive integer input, say n, the second column is a string of bits. What does that bitstring represent?

4. Each of these code fragments takes if $O(n \log n)$.time, but not necessarily $\Theta(n \log n)$. Give the asymptotic complexity of each in terms of n, using Θ in each case.

   (a)
   ```
   for(int i = 1; i < n; i++)
     for(int j = 1; j < i; j = 2*j);
      cout << "Hello" << endl;
   ```

   (b)
   ```
   for(int i = 1; i < n; i++)
     for(int j = i; j < n; j = 2*j);
      cout << "Hello" << endl;
   ```

   (c)
   ```
   for(int i = 1; i < n; i=2*i)
     for(int j = 1; j < i; j++);
      cout << "Hello" << endl;
   ```

   (d)
   ```
   for(int i = 1; i < n; i=2*i)
     for(int j = i; j < n; j++);
       cout << "Hello" << endl;
   ```

   (e)
   ```
   for(int i = n; i > 1; i=i/2)
     for(int j = i; j > 1; j--);
      cout << "Hello" << endl;
   ```

   (f)
   ```
   for(int i = n; i > 1; i=i/2)
     for(int j = n; j > i; j--);
      cout << "Hello" << endl;
   ```

5. These problems are harder than the ones above. Given the asymptotic complexity of each fragment in terms of n, using Θ.

   (g)
   ```
   for(int i = 1; i < n; i=2*i)
     for(int j = 1; j < i; j=2*j);
      cout << "Hello" << endl;
   ```
   Hint: Use substitution. Let `m = log n, k = log i, l = log j.`

   (h)
   ```
   for(int i = 2; i < n; i=i*i)
    cout << "Hello" << endl;
   ```
   Hint: Use substitution. Let `m = log n, k = log i.`

   (i)
   ```
   for(int i = 2; i < n; i=i*i)
     for(int j = 1; j < i; j = 2*j)
      cout << "Hello" << endl;
   ```
   Hint: Use substitution. Let `m = log n, k = log i, l = log j.`

   (j)
   ```
   for(int i = n; i > 1; i = log i)
    cout << "Hello" << endl;
   ```
   Hint: The answer is a function you've possibly never heard of. That function is defined on page 136 of the textbook. I will simply tell you the answer, and you will need to remember it.